A Simple Panel Unit-Root Test with Smooth Breaks in the Presence of a Multifactor Error Structure

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Review of Panel Unit Root Tests

- Consider the following *AR(1)* model (Dickey and Fuller, 1979):

$$y_t = \alpha + \rho y_{t-1} + \varepsilon_t,\ t = 1, 2, ..., T.$$

It can be equivalently written as

$$y_t - y_{t-1} = \alpha + (\rho - 1) y_{t-1} + \varepsilon_t,$$

or

$$\Delta y_t = \alpha + \beta y_{t-1} + \varepsilon_t,$$

The unit root hypothesis $\boldsymbol{\rho} = 1$ is equivalent to $\boldsymbol{\beta} = 0$.

Review of Panel Unit Root Tests

- From Levin et al. (2002):

  "...This paper considers pooling cross-*section time series data as a means of generating more powerful unit root tests."*

Review of Panel Unit Root Tests

- First Generation: Assume that idiosyncratic errors are cross-sectionally independent. Banerjee (1999); Levin et al., (2002); Im et al., (2003,IPS) and Maddala and Wu (1999).

$$\Delta y_{it} = \alpha_i + \beta_i y_{i,t-1} + \varepsilon_{it}, \quad t = 1, .., T; \quad i = 1, .., N,$$

where ε_{it} are independent for all *i* and *t*. The panel unit-root null hypothesis can be expressed as:

$$H_0 : \beta_i = 0, \quad \forall i$$

Review of Panel Unit Root Tests

- Second Generation: Allow idiosyncratic errors are cross-sectionally dependent. See the common factor models of Bai and Ng (2004); Moon and Perron (2004) and Pesaran (2007) and Pesaran et al. (2013).
  - De-factor method (Bai and Ng , 2004):

$$y_{it} = \alpha_i + \gamma'_{iy}\mathbf{f}_t + e_{it},$$

where

$$\begin{aligned}(1-L)\mathbf{f}_t &= C(L)\mathbf{u}_t,\\ (1-\phi_i L)e_{it} &= D_i(L)\varepsilon_{iyt},\end{aligned}$$

$$y_{it} \sim I(1) \text{ if } \phi_i = 1.$$

Review of Panel Unit Root Tests

- Cross-sectional mean approximation (Pesaran et al. , 2007, 2013):

$$\Delta y_{it} = \alpha_i + \beta_i y_{i,t-1} + \gamma'_{iy} \mathbf{f}_t + \varepsilon_{iyt}, \quad t = 1, .., T; \quad i = 1, .., N,$$

The unit-root hypothesis, $\phi_i = 1$ for all i, can be expressed as:

$$H_0 : \beta_i = 0, \quad \forall i$$

Review of Panel Unit Root Tests

- New development: Cross-dependent error and consideration of Structural change
  - The impact of structural change on traditional unit root test:
    - Low power (Spurious Accept ion): Perron (1989)
    - Over size (Spurious Rejection): Lebourney et al. (1998)

- Bai and Carrion-i-Silvestre (2009, RES): Adding Dummy variables at Bai and Ng(2004) 's model:

$$y_{it} = \delta_i' D_{i,t} + \gamma_{iy}' \mathbf{f}_t + e_{it},$$

where

$$\begin{aligned} (1-L)\mathbf{f}_t &= C(L)\mathbf{u}_t, \\ (1-\phi_i L)e_{it} &= D_i(L)\varepsilon_{iyt}, \end{aligned}$$

Review of Panel Unit Root Tests

- Im et al. (2010, Working paper): Adding Dummy variables

$$\Delta y_{it} = \delta_i' \Delta D_{i,t} + \beta_i y_{i,t-1} + \gamma_{iy}' \mathrm{f}_t + \varepsilon_{iyt}, \quad t = 1,..,T; \quad i = 1,..,N,$$

The unit-root hypothesis, $\phi_i = 1$ for all i, can be expressed as:

$$H_0 : \beta_i = 0, \quad \forall i$$

Fourier Form Change

- It is difficult to precisely estimate the number and magnitudes of multiple breaks.(Prodan, 2008).
- Fourier form break: Becker et al. (2004, 2006), Enders and Lee (2012)

$$d(t) = \alpha_0 + \sum_{k=1}^{n} \alpha_k \sin(2\pi kt/T) + \sum_{k=1}^{n} \beta_k \cos(2\pi kt/T) + \gamma \cdot t.$$

  - DGP is Fourier Form (Smooth Break)
  - Fourier Function is an approximation to Instantaneous break.

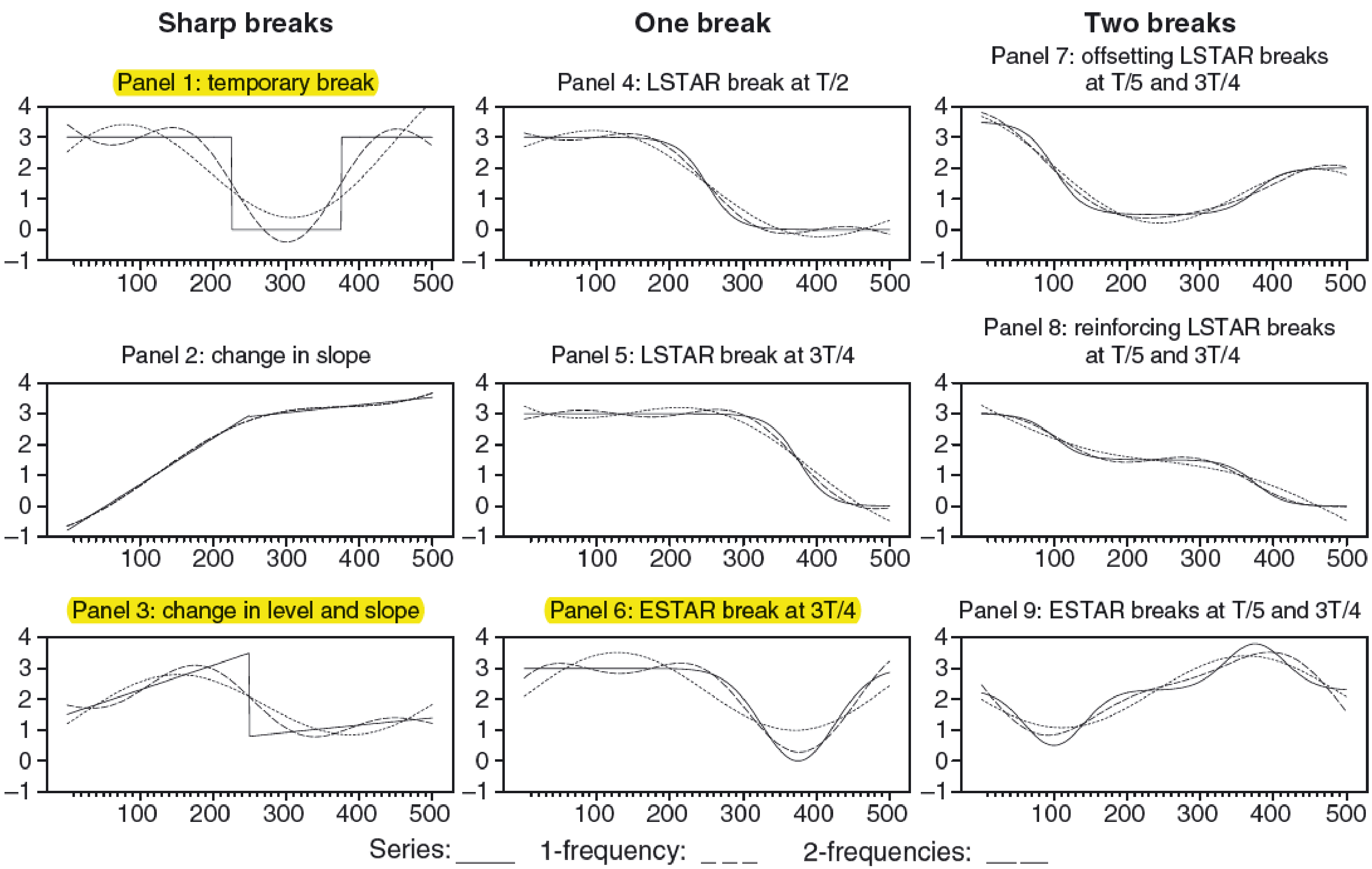

Figure 1. Sharp, ESTAR and LSTAR breaks

Our Model

- Model and Test statistics
  - Data Generating Process: A single frequency Fourier-form Break.

$$\Delta y_{it} = \beta_i y_{i,t-1} - \beta_i \boldsymbol{\alpha}'_{iy}\mathbf{d}_t + \phi_i \boldsymbol{\alpha}'_{iy}\Delta\mathbf{d}_t + \boldsymbol{\gamma}'_{iy}\mathbf{f}_t + \varepsilon_{iyt},$$

$$t = 1, .., T; \;\; i = 1, .., N,$$

where $\mathbf{d}_t = (1, \sin(2\pi\kappa t/T), \cos(2\pi\kappa t/T), t)'$.

The panel unit-root hypothesis can be expressed as:

$$H_0 : \beta_i = 0, \quad \forall i$$

against the possibly heterogeneous alternative,

$$H_1 : \beta_i < 0, \; i = 1, 2, ..., N_1; \; \beta_i = 0, \; i = N_1 + 1, N_1 + 2, ..., N.$$

Our Model

- (Individual) Breaks and cross dependence augmented Dickey-Fuller testing equation (BCADF):

$$\Delta y_{it} = c_{i,0} + \underline{c_{i,1}\sin(2\pi\kappa t/T) + c_{i,2}\cos(2\pi\kappa t/T)} + \mathbf{c}'_{i,3}\overline{\mathbf{z}}_{t-1} + \mathbf{c}'_{i,4}\Delta\overline{\mathbf{z}}_t + b_i y_{i,t-1} + e_{it}, \quad t = 1, 2, ..., T.$$

The t-statistic of the estimate of b_i ($\hat{b}_i$) is applied to examine the unit-root hypothesis and is expressed as:

$$t_i(N,T) = \frac{\Delta \mathbf{y}'_i \mathbf{M}_z \mathbf{y}_{i,-1}}{\hat{\sigma}_i(\mathbf{y}'_{i,-1}\mathbf{M}_z\mathbf{y}_{i,-1})^{1/2}},$$

Our Model

- Theoretical Results
  - Asymptotic Distribution of BCADF under the null hypothesis

Theorem 2.

$$BCADF_{if} \xrightarrow{(N,T)_{seq}} \frac{\int_0^1 W_i(r)dW_i(r) - \mathbf{q}'_{if}\boldsymbol{\Psi}_f^{-1}\mathbf{h}_{if}}{\left(\int_0^1 W_i^2(r)d(r) - \mathbf{h}'_{if}\boldsymbol{\Psi}_f^{-1}\mathbf{h}_{if}\right)^{1/2}}$$

Our Model

where

$$
\mathbf{q}_{if} = \begin{bmatrix} W_i(1) \\ -2\pi\kappa \int_0^1 \cos(2\pi\kappa r) W_i(r)dr \\ W(1) + 2\pi\kappa \int_0^1 \sin(2\pi\kappa r) W_i(r)dr \\ \int_0^1 [\mathbf{W}_{\mathbf{f}}(r)] dW_i(r) \end{bmatrix},
$$

$$
\mathbf{h}_{if} = \begin{bmatrix} \int_0^1 W_i(r)dr \\ -2\pi\kappa \left(\int_0^1 \cos(2\pi\kappa r) \left[\int_0^r W_i(s)ds \right] dr \right) \\ \int_0^1 W_i(s)ds + 2\pi\kappa \int_0^1 \sin(2\pi\kappa r) \left[\int_0^r W_i(s)ds \right] dr \\ \int_0^1 [\mathbf{W}_{\mathbf{f}}(r)] W_i(r)dr \end{bmatrix},
$$

$$
\mathbf{\Psi}_f = \begin{bmatrix} \mathbf{H}_{3\times 3} & \mathbf{R}_{3\times m} \\ \mathbf{R}'_{m\times 3} & \mathbf{J}_{m\times m} \end{bmatrix}, \ldots\ldots
$$

Theorem 2 shows that the asymptotic distribution of $t_i(N,T)$ depends only on the frequency parameter, κ , but is invariant to all other parameters in the DGP.

Our Model

- Pesaran et al. (2013)'s CADF under Fourier form break.

Theorem 3.

- T fixed, and $N \to \infty$:

  Let $t_i^{PSY,B}(N,T)$ be the statistic for testing the unit-root hypothesis when Fourier form breaks exist in the DGP.

$$t_i^{PSY,B}(N,T) \xrightarrow{N} \frac{\frac{\varepsilon'_{iy} s_{iy,-1}}{\sigma_i^2 T} - \mathring{\mathbf{q}}'_{iT} \Upsilon_{fT}^{-1} \mathring{\mathbf{h}}_{iT}}{J_1^{p*} \times J_2^{p*}} \oplus \frac{O(T^{-1/2})}{O(T^{-1/4})}$$

Our Model

Theorem 3.

- N, T all tend to infinity:

$$t_i^{PSY,B}(N,T) \xrightarrow{(N,T)_{seq}} \frac{\int_0^1 W_i(r)dW_i(r) - \boldsymbol{\omega}'_{i\mathbf{v}}\mathbf{G}_{\mathbf{v}}^{-1}\boldsymbol{\pi}_{i\mathbf{v}}}{\left(\int_0^1 W_i^2(r)dr - \boldsymbol{\pi}'_{i\mathbf{v}}\mathbf{G}_{\mathbf{v}}^{-1}\boldsymbol{\pi}_{i\mathbf{v}}\right)^{1/2}}$$

The expression $\frac{\int_0^1 W_i(r)dW_i(r) - \boldsymbol{\omega}'_{i\mathbf{v}}\mathbf{G}_{\mathbf{v}}^{-1}\boldsymbol{\pi}_{i\mathbf{v}}}{\left(\int_0^1 W_i^2(r)dr - \boldsymbol{\pi}'_{i\mathbf{v}}\mathbf{G}_{\mathbf{v}}^{-1}\boldsymbol{\pi}_{i\mathbf{v}}\right)^{1/2}}$ is the same as the limiting distribution of the *CADF* statistic proposed by Pesaran et al. (2013, Theorem 2.1) when there is no break in the DGP.

Our Model

- BCADF-based Panel Unit-Root Tests

$$BCIPS(N,T) = \frac{1}{N}\sum_{i=1}^{N} t_i(N,T),$$

  - Critical values.

- Extension to serially correlated errors

$$\begin{aligned}\Delta y_{it} &= c_{i,0} + c_{i,1}\sin(2\pi\kappa t/T) + c_{i,2}\cos(2\pi\kappa t/T) + \mathbf{c}'_{i,3}\overline{\mathbf{z}}_{t-1} \\ &+ \mathbf{c}'_{i,4}\Delta\overline{\mathbf{z}}_t + \sum_{j=1}^{p}\mathbf{c}'_{i,5,j}\Delta\overline{\mathbf{z}}_{t-j} + \sum_{j=1}^{p} c_{i,6,j}\Delta y_{i,t-j} \\ &+ b_i y_{i,t-1} + e_{it}, \quad t = 1, 2, ..., T\end{aligned}$$

Table B1. Critical values of the *BCIPS* test with m=1 – with an intercept only

		K=1					K=2					K=3					K=4					K=5				
p	$T \setminus N$	20	30	50	100	200	20	30	50	100	200	20	30	50	100	200	20	30	50	100	200	20	30	50	100	200
1%																										
0	50	-3.22	-3.14	-3.07	-3.03	-3.00	-2.75	-2.65	-2.59	-2.52	-2.48	-2.50	-2.43	-2.35	-2.28	-2.25	-2.40	-2.33	-2.25	-2.19	-2.16	-2.36	-2.29	-2.21	-2.15	-2.12
	70	-3.20	-3.13	-3.08	-3.02	-3.00	-2.75	-2.66	-2.59	-2.53	-2.49	-2.51	-2.44	-2.37	-2.30	-2.27	-2.42	-2.35	-2.28	-2.22	-2.19	-2.38	-2.31	-2.24	-2.18	-2.15
	100	-3.20	-3.13	-3.07	-3.03	-3.00	-2.75	-2.67	-2.59	-2.53	-2.49	-2.52	-2.45	-2.38	-2.31	-2.29	-2.44	-2.36	-2.30	-2.24	-2.21	-2.40	-2.32	-2.26	-2.20	-2.17
	200	-3.19	-3.13	-3.07	-3.03	-3.00	-2.75	-2.67	-2.60	-2.53	-2.50	-2.54	-2.46	-2.39	-2.34	-2.30	-2.45	-2.38	-2.32	-2.26	-2.22	-2.41	-2.34	-2.29	-2.22	-2.19
1	50	-3.22	-3.15	-3.08	-3.04	-3.01	-2.73	-2.63	-2.57	-2.50	-2.46	-2.44	-2.36	-2.28	-2.21	-2.17	-2.32	-2.25	-2.17	-2.11	-2.07	-2.27	-2.20	-2.12	-2.07	-2.03
	70	-3.20	-3.14	-3.08	-3.03	-3.00	-2.74	-2.64	-2.57	-2.51	-2.47	-2.47	-2.39	-2.32	-2.25	-2.22	-2.36	-2.29	-2.22	-2.16	-2.12	-2.32	-2.24	-2.18	-2.12	-2.09
	100	-3.21	-3.13	-3.07	-3.03	-3.00	-2.73	-2.65	-2.58	-2.52	-2.48	-2.49	-2.42	-2.35	-2.28	-2.25	-2.41	-2.32	-2.26	-2.20	-2.17	-2.36	-2.28	-2.22	-2.16	-2.12
	200	-3.19	-3.13	-3.07	-3.03	-3.00	-2.74	-2.66	-2.59	-2.53	-2.49	-2.52	-2.45	-2.38	-2.32	-2.29	-2.43	-2.36	-2.30	-2.24	-2.21	-2.39	-2.32	-2.26	-2.20	-2.17
2	50	-3.15	-3.08	-3.02	-2.97	-2.94	-2.66	-2.56	-2.49	-2.42	-2.38	-2.33	-2.25	-2.16	-2.09	-2.05	-2.20	-2.13	-2.05	-1.99	-1.96	-2.17	-2.10	-2.02	-1.96	-1.93
	70	-3.16	-3.10	-3.04	-2.98	-2.96	-2.69	-2.59	-2.52	-2.45	-2.41	-2.39	-2.31	-2.24	-2.17	-2.13	-2.28	-2.20	-2.13	-2.07	-2.04	-2.24	-2.15	-2.10	-2.04	-2.00
	100	-3.17	-3.10	-3.04	-3.00	-2.97	-2.69	-2.61	-2.54	-2.48	-2.44	-2.44	-2.37	-2.29	-2.22	-2.19	-2.35	-2.26	-2.19	-2.14	-2.10	-2.30	-2.22	-2.15	-2.10	-2.06
	200	-3.17	-3.11	-3.05	-3.02	-2.99	-2.72	-2.64	-2.58	-2.51	-2.47	-2.50	-2.43	-2.35	-2.29	-2.26	-2.41	-2.34	-2.27	-2.21	-2.18	-2.37	-2.29	-2.23	-2.17	-2.14
3	50	-3.16	-3.08	-3.01	-2.96	-2.93	-2.63	-2.53	-2.46	-2.39	-2.36	-2.27	-2.17	-2.09	-2.02	-1.98	-2.15	-2.08	-2.00	-1.93	-1.90	-2.16	-2.08	-2.01	-1.95	-1.91
	70	-3.16	-3.10	-3.04	-2.98	-2.96	-2.67	-2.57	-2.50	-2.43	-2.39	-2.35	-2.26	-2.18	-2.11	-2.08	-2.23	-2.16	-2.08	-2.02	-1.98	-2.20	-2.11	-2.06	-2.00	-1.96
	100	-3.18	-3.10	-3.05	-3.00	-2.97	-2.68	-2.61	-2.53	-2.46	-2.42	-2.40	-2.33	-2.25	-2.19	-2.15	-2.30	-2.22	-2.15	-2.09	-2.06	-2.26	-2.18	-2.11	-2.06	-2.02
	200	-3.17	-3.12	-3.06	-3.02	-2.99	-2.72	-2.63	-2.57	-2.50	-2.46	-2.48	-2.41	-2.34	-2.28	-2.24	-2.39	-2.32	-2.25	-2.19	-2.16	-2.34	-2.27	-2.21	-2.15	-2.12
4	50	-3.07	-3.00	-2.93	-2.87	-2.85	-2.57	-2.47	-2.38	-2.32	-2.27	-2.16	-2.08	-1.99	-1.92	-1.88	-2.11	-2.02	-1.94	-1.88	-1.84	-2.16	-2.07	-2.00	-1.94	-1.90
	70	-3.12	-3.04	-2.98	-2.94	-2.91	-2.61	-2.52	-2.45	-2.37	-2.34	-2.27	-2.18	-2.10	-2.04	-1.99	-2.17	-2.09	-2.01	-1.95	-1.91	-2.16	-2.08	-2.01	-1.95	-1.91
	100	-3.14	-3.07	-3.01	-2.97	-2.94	-2.66	-2.58	-2.50	-2.42	-2.39	-2.36	-2.27	-2.20	-2.13	-2.10	-2.24	-2.17	-2.10	-2.03	-2.00	-2.21	-2.13	-2.06	-2.01	-1.97
	200	-3.17	-3.10	-3.04	-3.00	-2.97	-2.69	-2.62	-2.55	-2.48	-2.45	-2.46	-2.37	-2.31	-2.24	-2.20	-2.35	-2.29	-2.22	-2.16	-2.12	-2.32	-2.24	-2.18	-2.12	-2.08
5%																										
0	50	-3.04	-2.99	-2.95	-2.92	-2.91	-2.55	-2.49	-2.44	-2.40	-2.38	-2.32	-2.27	-2.22	-2.18	-2.16	-2.23	-2.18	-2.13	-2.09	-2.08	-2.18	-2.14	-2.09	-2.06	-2.04
	70	-3.03	-2.99	-2.95	-2.92	-2.91	-2.55	-2.50	-2.45	-2.41	-2.39	-2.33	-2.28	-2.24	-2.20	-2.18	-2.25	-2.20	-2.16	-2.12	-2.10	-2.21	-2.16	-2.12	-2.09	-2.07
	100	-3.03	-2.98	-2.95	-2.92	-2.91	-2.55	-2.50	-2.46	-2.42	-2.40	-2.34	-2.30	-2.25	-2.22	-2.20	-2.27	-2.22	-2.18	-2.14	-2.12	-2.23	-2.18	-2.14	-2.11	-2.09
	200	-3.02	-2.98	-2.95	-2.92	-2.91	-2.56	-2.50	-2.46	-2.43	-2.41	-2.36	-2.31	-2.27	-2.23	-2.22	-2.29	-2.24	-2.20	-2.16	-2.15	-2.25	-2.20	-2.17	-2.13	-2.11
1	50	-3.04	-2.99	-2.95	-2.92	-2.91	-2.52	-2.45	-2.41	-2.37	-2.35	-2.25	-2.19	-2.14	-2.10	-2.08	-2.15	-2.09	-2.04	-2.00	-1.99	-2.10	-2.05	-2.00	-1.97	-1.95
	70	-3.03	-2.99	-2.95	-2.92	-2.91	-2.53	-2.47	-2.43	-2.39	-2.37	-2.28	-2.23	-2.19	-2.15	-2.13	-2.19	-2.14	-2.10	-2.06	-2.04	-2.15	-2.10	-2.06	-2.02	-2.00
	100	-3.03	-2.98	-2.95	-2.93	-2.91	-2.54	-2.48	-2.44	-2.41	-2.38	-2.31	-2.26	-2.22	-2.18	-2.16	-2.23	-2.18	-2.14	-2.10	-2.08	-2.19	-2.14	-2.10	-2.06	-2.04
	200	-3.02	-2.99	-2.95	-2.92	-2.91	-2.55	-2.50	-2.46	-2.42	-2.40	-2.35	-2.30	-2.25	-2.22	-2.20	-2.27	-2.22	-2.18	-2.14	-2.13	-2.23	-2.18	-2.14	-2.11	-2.09
2	50	-2.97	-2.92	-2.88	-2.85	-2.83	-2.44	-2.37	-2.32	-2.28	-2.25	-2.13	-2.07	-2.02	-1.98	-1.95	-2.03	-1.97	-1.93	-1.88	-1.86	-2.00	-1.94	-1.90	-1.86	-1.84
	70	-2.99	-2.94	-2.91	-2.88	-2.86	-2.47	-2.41	-2.36	-2.32	-2.30	-2.20	-2.15	-2.10	-2.06	-2.04	-2.10	-2.06	-2.00	-1.97	-1.95	-2.07	-2.01	-1.97	-1.93	-1.92
	100	-3.00	-2.95	-2.92	-2.89	-2.88	-2.49	-2.45	-2.40	-2.36	-2.34	-2.25	-2.21	-2.16	-2.12	-2.10	-2.17	-2.11	-2.07	-2.04	-2.02	-2.13	-2.08	-2.03	-2.00	-1.98
	200	-3.01	-2.97	-2.93	-2.91	-2.90	-2.53	-2.48	-2.43	-2.40	-2.38	-2.32	-2.27	-2.22	-2.19	-2.17	-2.24	-2.19	-2.15	-2.11	-2.09	-2.20	-2.15	-2.11	-2.07	-2.06
3	50	-2.96	-2.91	-2.87	-2.83	-2.82	-2.40	-2.33	-2.28	-2.23	-2.21	-2.07	-2.00	-1.95	-1.90	-1.87	-1.97	-1.92	-1.87	-1.82	-1.80	-1.98	-1.93	-1.87	-1.83	-1.81
	70	-2.99	-2.94	-2.90	-2.87	-2.85	-2.45	-2.38	-2.34	-2.29	-2.27	-2.15	-2.09	-2.04	-2.00	-1.98	-2.05	-2.00	-1.95	-1.91	-1.89	-2.03	-1.97	-1.93	-1.89	-1.87
	100	-3.00	-2.95	-2.91	-2.89	-2.87	-2.48	-2.43	-2.38	-2.34	-2.32	-2.22	-2.17	-2.12	-2.08	-2.06	-2.13	-2.07	-2.03	-1.99	-1.97	-2.09	-2.04	-1.99	-1.96	-1.94
	200	-3.01	-2.97	-2.93	-2.91	-2.90	-2.53	-2.47	-2.43	-2.39	-2.37	-2.30	-2.25	-2.21	-2.17	-2.15	-2.22	-2.17	-2.12	-2.09	-2.07	-2.18	-2.13	-2.09	-2.05	-2.03
4	50	-2.87	-2.82	-2.78	-2.74	-2.73	-2.31	-2.25	-2.19	-2.14	-2.11	-1.96	-1.90	-1.84	-1.79	-1.77	-1.92	-1.86	-1.81	-1.76	-1.74	-1.96	-1.91	-1.86	-1.82	-1.80
	70	-2.93	-2.88	-2.84	-2.81	-2.80	-2.38	-2.33	-2.27	-2.23	-2.20	-2.07	-2.02	-1.96	-1.92	-1.89	-1.99	-1.93	-1.88	-1.84	-1.82	-1.98	-1.93	-1.88	-1.84	-1.82
	100	-2.96	-2.92	-2.88	-2.85	-2.84	-2.44	-2.39	-2.34	-2.29	-2.27	-2.16	-2.11	-2.06	-2.02	-2.00	-2.07	-2.01	-1.97	-1.93	-1.91	-2.04	-1.99	-1.94	-1.91	-1.88
	200	-3.00	-2.96	-2.92	-2.89	-2.88	-2.50	-2.45	-2.41	-2.37	-2.35	-2.27	-2.23	-2.18	-2.14	-2.12	-2.19	-2.14	-2.10	-2.06	-2.04	-2.15	-2.10	-2.06	-2.02	-2.00

Our Model

- BCADF-based Panel Unit-Root Tests

$$BCIPS(N,T) = \frac{1}{N}\sum_{i=1}^{N} t_i(N,T),$$

  - Critical values.

- Extension to serially correlated errors

$$\begin{aligned}
\Delta y_{it} &= c_{i,0} + c_{i,1}\sin(2\pi\kappa t/T) + c_{i,2}\cos(2\pi\kappa t/T) + \mathbf{c}'_{i,3}\overline{\mathbf{z}}_{t-1} \\
&\quad + \mathbf{c}'_{i,4}\Delta\overline{\mathbf{z}}_t + \sum_{j=1}^{p}\mathbf{c}'_{i,5,j}\Delta\overline{\mathbf{z}}_{t-j} + \sum_{j=1}^{p} c_{i,6,j}\Delta y_{i,t-j} \\
&\quad + b_i y_{i,t-1} + e_{it}, \quad t = 1, 2, ..., T
\end{aligned}$$

Our Model

- Finite sample performance

  - Size and Power of BCIPS.
  - Size of Pesaran et al.'s (2013) CIPS.

Table 1. Sizes and Powers of the *BCIPS* test with two known factors (m=2) in which factors and idiosyncratic errors are serially uncorrelated –with an Intercept only

T \ N	$\kappa=1$					$\kappa=2$					$\kappa=3$				
	20	30	50	100	200	20	30	50	100	200	20	30	50	100	200
								Size							
						$BCIPS(\hat{p},\kappa)$, $\alpha_{iy,1},\alpha_{iy,2}$~i.i.d.$U[1,2]$, $\alpha_{ix,1},\alpha_{ix,2}$~i.i.d.$U[1,2]$									
50	0.053	0.039	0.048	0.045	0.044	0.044	0.050	0.056	0.046	0.047	0.053	0.054	0.043	0.052	0.048
70	0.049	0.038	0.044	0.042	0.052	0.045	0.040	0.045	0.046	0.055	0.048	0.053	0.053	0.050	0.044
100	0.047	0.050	0.038	0.046	0.049	0.046	0.049	0.051	0.045	0.039	0.038	0.054	0.045	0.048	0.047
200	0.056	0.034	0.047	0.044	0.043	0.042	0.047	0.046	0.046	0.047	0.048	0.044	0.049	0.052	0.051
						$BCIPS(\hat{p},\kappa)$, $\alpha_{iy,1},-\alpha_{iy,2}\sim$ i.i.d.$U[10,20]$, $-\alpha_{ix,1}$, $\alpha_{ix,2}\sim$ i.i.d.$U[3,5]$									
50	0.047	0.056	0.048	0.042	0.060	0.054	0.050	0.048	0.048	0.047	0.054	0.049	0.046	0.045	0.046
70	0.043	0.056	0.049	0.050	0.053	0.048	0.056	0.047	0.044	0.057	0.050	0.046	0.055	0.048	0.051
100	0.038	0.039	0.046	0.051	0.048	0.047	0.046	0.050	0.049	0.056	0.056	0.043	0.052	0.051	0.052
200	0.051	0.046	0.043	0.047	0.052	0.047	0.051	0.046	0.044	0.047	0.039	0.053	0.052	0.043	0.047
						$BCIPS(\hat{p},\kappa)$, $\alpha_{iy,1},\alpha_{iy,2}\sim$ i.i.d.$U[10,100]$, $\alpha_{ix,1}$, $\alpha_{ix,2}\sim$ i.i.d.$U[3,5]$									
50	0.041	0.050	0.046	0.050	0.042	0.045	0.049	0.060	0.068	0.054	0.057	0.045	0.047	0.048	0.049
70	0.044	0.039	0.055	0.057	0.054	0.053	0.044	0.046	0.045	0.046	0.047	0.059	0.054	0.053	0.052
100	0.049	0.049	0.040	0.045	0.055	0.047	0.044	0.051	0.045	0.039	0.041	0.040	0.053	0.047	0.045
200	0.051	0.040	0.048	0.047	0.043	0.053	0.046	0.054	0.051	0.048	0.063	0.045	0.045	0.050	0.048
								Power							
						$BCIPS(\hat{p},\kappa)$, $\alpha_{iy,1},\alpha_{iy,2}$~i.i.d.$U[1,2]$, $\alpha_{ix,1},\alpha_{ix,2}$~i.i.d.$U[1,2]$									
50	0.406	0.436	0.541	0.546	0.618	0.556	0.555	0.629	0.615	0.662	0.662	0.651	0.729	0.799	0.846
70	0.579	0.752	0.793	0.924	0.930	0.736	0.866	0.910	0.955	0.957	0.830	0.943	0.975	0.996	0.997
100	0.881	0.983	0.980	1.000	1.000	0.967	0.998	0.998	1.000	1.000	0.990	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
						$BCIPS(\hat{p},\kappa)$, $\alpha_{iy,1},-\alpha_{iy,2}\sim$ i.i.d.$U[10,20]$, $-\alpha_{ix,1}$, $\alpha_{ix,2}\sim$ i.i.d.$U[3,5]$									
50	0.367	0.492	0.559	0.654	0.764	0.481	0.560	0.632	0.676	0.700	0.581	0.707	0.764	0.866	0.926
70	0.549	0.756	0.815	0.956	0.988	0.725	0.901	0.921	0.979	0.995	0.866	0.966	0.988	0.999	1.000
100	0.969	0.941	0.998	1.000	1.000	0.997	0.985	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
						$BCIPS(\hat{p},\kappa)$, $\alpha_{iy,1},\alpha_{iy,2}\sim$ i.i.d.$U[10,100]$, $\alpha_{ix,1}$, $\alpha_{ix,2}\sim$ i.i.d.$U[3,5]$									
50	0.218	0.354	0.319	0.284	0.301	0.382	0.603	0.659	0.697	0.693	0.598	0.819	0.906	0.969	0.990
70	0.466	0.617	0.692	0.873	0.949	0.841	0.937	0.965	0.998	1.000	0.950	0.990	0.999	1.000	1.000
100	0.966	0.987	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Our Model

- Finite sample performance
  - Size and Power of BCIPS.
  - Size of Pesaran et al.'s (2013) CIPS.

Table 2. Sizes of Pesaran et al.'s (2013) CIPS test with two known factors in which factors and idiosyncratic errors are serially uncorrelated –with an Intercept only

T \ N	$\kappa=1$					$\kappa=2$					$\kappa=3$				
	20	30	50	100	200	20	30	50	100	200	20	30	50	100	200
Size: Pesaran's $CIPS(\hat{p}, \kappa)$, $\alpha_{iy,1}, -\alpha_{iy,2}$~i.i.d.$U(1, 2)$, $-\alpha_{ix,1}$, $\alpha_{ix,2}$ ~ i.i.d.$U(1,2)$															
50	0.166	0.169	0.205	0.226	0.252	0.073	0.069	0.069	0.088	0.078	0.022	0.018	0.013	0.008	0.007
70	0.104	0.117	0.140	0.171	0.189	0.050	0.053	0.048	0.069	0.062	0.014	0.010	0.013	0.013	0.007
100	0.084	0.086	0.121	0.145	0.123	0.042	0.039	0.057	0.052	0.050	0.021	0.011	0.022	0.014	0.010
200	0.074	0.077	0.069	0.076	0.085	0.034	0.045	0.032	0.027	0.023	0.023	0.019	0.015	0.005	0.006
Size: Pesaran's $CIPS(\hat{p}, \kappa)$, $\alpha_{iy,1}, -\alpha_{iy,2}$~i.i.d.$U(3, 5)$, $-\alpha_{ix,1}$, $\alpha_{ix,2}$ ~ i.i.d.$U(3,5)$															
50	0.373	0.418	0.428	0.523	0.574	0.104	0.089	0.083	0.105	0.109	0.021	0.013	0.007	0.012	0.005
70	0.257	0.312	0.328	0.410	0.466	0.072	0.070	0.083	0.089	0.099	0.012	0.013	0.010	0.013	0.006
100	0.204	0.274	0.280	0.327	0.355	0.060	0.113	0.073	0.090	0.105	0.019	0.026	0.015	0.012	0.014
200	0.197	0.159	0.204	0.240	0.237	0.119	0.057	0.097	0.093	0.099	0.053	0.016	0.030	0.030	0.022
Size: Pesaran's $CIPS(\hat{p}, \kappa)$, $\alpha_{iy,1}, -\alpha_{iy,2}$~i.i.d.$U(10, 20)$, $-\alpha_{ix,1}$, $\alpha_{ix,2}$ ~ i.i.d.$U(3,5)$															
50	0.338	0.341	0.620	0.595	0.717	0.064	0.072	0.105	0.094	0.125	0.014	0.013	0.010	0.010	0.006
70	0.309	0.399	0.571	0.447	0.515	0.056	0.078	0.092	0.080	0.065	0.013	0.014	0.014	0.007	0.003
100	0.165	0.364	0.309	0.421	0.518	0.038	0.094	0.041	0.063	0.101	0.010	0.024	0.008	0.008	0.010
200	0.156	0.403	0.289	0.375	0.474	0.039	0.194	0.086	0.148	0.148	0.012	0.052	0.018	0.034	0.028

Our Model

- Empirical example: Examines the validity of long-run PPP by testing the stationarity of real exchange rates.
  - Cross-sectional unit: 30 OECD countries (N = 30).
  - Time series period: 1981Q1-2011Q4 (T = 124).
  - Empirical Results: We therefore conclude that there is little evidence to support long-run PPP.

Table 6. The *BCIPS* and *CIPS* panel unit-root tests for real exchange rates

$$\Delta q_{it} = c_{i,0} + c_{i,1}\sin(2\pi\kappa t/T) + c_{i,2}\cos(2\pi\kappa t/T)) + \mathbf{c}'_{i,3}\overline{\mathbf{z}}_{t-1} + \mathbf{c}'_{i,4}\Delta\overline{\mathbf{z}}_t + \sum_{j=1}^{p}\mathbf{c}'_{i,5,j}\Delta\overline{\mathbf{z}}_{t-j}$$
$$+\sum_{j=1}^{p} c_{i,6,j}\Delta q_{i,t-j} + b_i q_{i,t-1} + e_{it}, \quad \text{where } \mathbf{z}_{it} = (q_{it}, \mathbf{x}'_{it})'.$$

Included $\mathbf{x}_{it}$	$(\hat{p},\hat{\kappa})$	$[N,T]$	*CD*	$\boxed{BCIPS}$	$\boxed{CIPS}$
		$\hat{p}$ is determined by the SBC rule in (39), m=1			
No	(1,1)	[29,124]	**116.7**	**-3.390****	-2.108
		m=2			
$\overline{gdp}$	(1,1)	[19,124]	**83.8**	**-3.757****	**-2.867****
p_{oil}	(1,1)	[29,124]	**116.7**	**-3.228***	-2.116
$\overline{r}^L$	(1,1)	[20,124]	**98.5**	**-3.331***	**-2.658****
$\overline{pd}$	(1,1)	[16,124]	**74.3**	-3.245	**-2.752****
		m=3			
$\overline{gdp}$, p_{oil}	(1,1)	[19,124]	**83.8**	**-3.510***	**-2.993****
p_{oil}, $\overline{r}^L$	(1,1)	[20,124]	**98.5**	-3.048	**-2.709***
$\overline{r}^L$, $\overline{gdp}$	(1,1)	[17,124]	**82.0**	**-3.701****	**-2.936****
$\overline{pd}$, $\overline{gdp}$	(1,1)	[15,124]	**68.6**	**-3.770****	**-3.418****
$\overline{pd}$, p_{oil}	(1,1)	[16,124]	**74.3**	-3.015	**-2.749***
$\overline{pd}$, $\overline{r}^L$	(1,1)	[15,124]	**68.6**	-3.206	**-2.781***
		m=4			
$\overline{gdp}$, p_{oil}, $\overline{r}^L$	(1,1)	[17,124]	**82.0**	-3.458	**-2.918***
$\overline{pd}$, p_{oil}, $\overline{r}^L$	(1,1)	[15,124]	**68.6**	-2.974	-2.713
$\overline{gdp}$, $\overline{pd}$, $\overline{r}^L$	(1,1)	[15,124]	**68.6**	-3.495	**-3.309****
$\overline{gdp}$, p_{oil}, $\overline{pd}$	(1,1)	[15,124]	**68.6**	**-3.775****	**-3.367****

Note: m is the number of factors in the model. *CD* is the cross-sectional dependence test of Pesaran (2004). The Bold faced numbers indicate significance at the 5% level. '**' indicates significance at the 1% level and '*' indicates significance at the 5% level . $\hat{\kappa}$ and $\hat{p}$ are jointly determined based on the rule of minimum sum of square described in section 3.6.

Our Model

- Conclusion: It is fair to say that the BCIPS test complements the panel unit-root tests using dummy variables.