PURCHASING POWER PARITY, PRODUCTIVITY DIFFERENTIALS AND NON-LINEARITY*

by

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The purpose of this paper is to apply a symmetric band threshold autoregressive model to investigate the non-linear adjustment of the real pound-dollar rate over a period from 1885 to 2003. After controlling for the Harrod–Balassa–Samuelson effects, we find evidence to support a non-linear mean reversion of the real pound–dollar rate. Moreover, the estimated half-life is about two years with large shocks. We therefore provide a solution to the purchasing power parity puzzle.

1 INTRODUCTION

Purchasing power parity (PPP) has been one of the most intensive research issues in empirical international finance because it is a cornerstone of many theoretical models in international economics. Empirically, the validity of PPP can be examined by testing the stationarity of real exchange rates. Conventional literature, based on linear unit-root tests, fails to reject the unit-root hypothesis for real exchange rates (Mark, 1990). Several authors argue that the failure of rejecting the unit-root hypothesis for real exchange rates may be due to the available short-span data from the post-Bretton-Woods period which results in the low power of unit-root tests (Shiller and Perron, 1985). Many studies apply panel unitroot tests to examine the stationarity of real exchange rates based on data from the post-Bretton-Woods period (Abuaf and Jorion, 1990; O'Connell,

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1998).¹ However, findings from panel tests regarding the stationarity of real exchange rates are mixed. An alternative strategy for PPP analysis is to apply long-span data in empirical analysis. Applying long-span data over two centuries on the dollar–sterling and franc–sterling real exchange rates, Lothian and Taylor (1996) find significant evidence of mean reversion of the real exchange rates. However, Engel and Kim (1999) find evidence of a permanent component in the US–UK real exchange rate over the period 1885–1994.

It is puzzling to researchers that the arbitrage in the goods market fails to eliminate a wedge between prices across countries given the fact that there is a huge range of goods traded internationally. There are two likely theoretical arguments that explain this phenomenon. The first relies on the so-called Harrod–Balassa–Samuelson (hereafter HBS) effect (Harrod, 1939; Balassa, 1964; Samuelson, 1964). The HBS effect suggests that, under some assumptions, fast-growing economies will experience a rising relative price of non-tradables and hence a real appreciation over time. In this case, deviations from PPP will revert to an equilibrium trend instead of a constant equilibrium. Based on the idea of differential productivity growth in tradables and non-tradables, Obstfeld (1993) develops a simple stochastic model in which real exchange rates contain a pronounced deterministic trend. There are several empirical studies supporting the existence of the HBS effect (Canzoneri *et al.*, 1999; Deloach, 2001). Therefore, the existence of the HBS effect may invalidate the conventional analysis with a constant equilibrium.

The second argument comes from recent theoretical analyses that demonstrate how transaction costs or sunk costs of international arbitrage result in a non-linear adjustment of financial variables (Dixit, 1989; Sercu *et al.*, 1995). International arbitrage moves deviations from PPP towards the equilibrium when the deviations are large relative to the transaction costs. However, there is no international arbitrage when deviations from PPP are smaller than the transaction costs and hence the deviations may move with a non-stationary process. In other words, deviations from PPP are shown to follow a non-linear process with the speed of adjustment towards the equilibrium varying directly with the extent of deviations from PPP. Some previous studies assume that the equilibrium real exchange rate is a constant and find support for the non-linear adjustment of real exchange rates (Michael *et al.*, 1997; Obstfeld and Taylor, 1997; Sarno *et al.*, 2004).

It is reasonable to argue that the equilibrium real exchange rate varies over time as a result of the HBS effect when the data over longer periods such as a century are applied. There is much evidence pointing out the importance of considering the HBS effect in modeling real exchange rates for long

¹Panel unit-root tests exploit cross-sectional and time series information which result in higher power relative to conventional unit-root tests.

historical data over a century (Cuddington and Liang, 2000; Lothian and Taylor, 2000, 2008; Taylor, 2002; Peel and Venetis, 2003). For example, Peel and Venetis (2003) and Lothian and Taylor (2008) apply an exponential smooth transition autoregressive (ESTAR) model to investigate the non-linear adjustment of the real exchange rate towards a non-constant equilibrium.

In this paper we focus on a band threshold autoregressive (band-TAR) model for several reasons. First, almost all of the existing literature using ESTAR-type models to describe real exchange rate dynamics assumes that real exchange rates are stationary without rigorous testing (Taylor *et al.*, 2001; Kilian and Taylor, 2003). In our paper, the unit-root hypothesis of real exchange rates against a non-linear TAR stationary process is empirically rejected based on the unit-root test provided by Bec *et al.* (2004). Second, in addition to support the contention that real exchange rates are TAR-type stationary, we provide evidence to reject the null hypothesis of linearity against TAR-type non-linearity. Third, a band-TAR model allows us to the measure of transaction costs (Imbs *et al.*, 2003). However, the ESTAR model focuses on smooth transitions between regimes and hence the model is not able to provide enough information on the width of the no-arbitrage band.

The purpose of this paper is to re-examine the adjustment speed of the real pound-dollar rate over the period 1885-2003 in which we take the HBS effect and the non-linear adjustment of the real rate into account. The HBS effect in our paper is measured by real GDP per capita difference. The rationale to select the period 1885-2003 is that Engel and Kim (1999) find evidence of a permanent component for the real pound-dollar rate over a similar period. We first provide evidence to show that the real pound-dollar rate is stationary but follows a symmetric band-TAR model with a nonconstant equilibrium and with a unit root in the middle regime. Moreover, we find that a non-linear trend approximation to the time-varying equilibrium of real exchange rates is crucial to the stationarity of the real pound-dollar rate over the period 1885–2003. The empirical methods we adopted are competent methods in analyzing threshold models based on an asymptotic econometric theorem. Next, the generalized impulse response function (GIRF) is applied to construct the half-life of the real pound-dollar rate. We find that the real pound-dollar rate reverts to equilibrium at a faster speed with a half-life of two years when shocks are large. Therefore, our findings provide a solution to the PPP puzzle outlined in Rogoff (1996).

The next section describes the band-TAR model considered in our empirical analysis and discusses its estimation and testing methodology. In Section 3, we describe estimation results and the half-life estimation of real exchange rates based on a GIRF. Finally, concluding remarks are given in Section 4.

2 The Econometric Model and its Estimation Methods

2.1 Model Estimation and Testing

We adopt a generalized band-TAR model provided by Balke and Fomby (1997) to estimate the non-linear mean-reverting behavior of real exchange rates. Let s_t be the nominal exchange rate (foreign currency per US dollar), p_t^* be the foreign price index, and p_t be the domestic price index. All of the variables are in logarithmic form. The real exchange rate is then defined as $s_t - p_t^* + p_t$. The band-TAR model for a non-constant equilibrium can be written as follows:

$$q_{t} - \overline{q}_{t} = \begin{cases} -\kappa + \sum_{i=1}^{m} \alpha_{i} (q_{t-i} - \overline{q}_{t-i} + \kappa) + \varepsilon_{t} & \text{if } q_{t-d} - \overline{q}_{t-d} \leq -\kappa \\ \sum_{i=1}^{m} \beta_{i} (q_{t-i} - \overline{q}_{t-i}) + \varepsilon_{t} & \text{if } |q_{t-d} - \overline{q}_{t-d}| < \kappa \\ \kappa + \sum_{i=1}^{m} \alpha_{i} (q_{t-i} - \overline{q}_{t-i} - \kappa) + \varepsilon_{t} & \text{if } q_{t-d} - \overline{q}_{t-d} \geq \kappa \end{cases}$$
(1)

where q_t is the real exchange rate, \overline{q}_t is the equilibrium of q_t , m is the lag order of the model and q_{t-d} is the threshold variable with d chosen among 1, 2, ..., m. The error terms ε_t are assumed to be independently and identically normally distributed with a zero mean and a constant variance σ^2 .

We can view the fluctuations in $q_t - \overline{q}_t$ as short-run 'disequilibrium' fluctuations around the long-run equilibrium \overline{q}_t . When the deviations of the real pound-dollar rate are smaller than the absolute value of the bandwidth κ (i.e. $|q_{t-d} - \overline{q}_{t-d}| < \kappa$), we note that the deviations are within the band and they follow an autoregressive (AR) process with parameters β_i , $i = 0, 1, \ldots$, m. Otherwise, the deviations are outside of the band and follow an alternative AR process. If the equilibrium real exchange rate is a constant, we set $\overline{q}_{i} = c$. Because long-term data are used in this paper and the USA transformed itself from an exclusively rural economy to replace Great Britain as the leading international economic power, it is likely that the productivity differential between these two countries, and hence the HBS effect, is significant during the period of investigation. A simple and conventional approximation for the HBS effect is to include a linear time trend in the model. Lothian and Taylor (2000) point out the significance of a non-linear trend in proximity to the HBS effect. Theoretically, the HBS effect arises from productivity differentials between the home and foreign countries. We measure the level of productivity by real GDP per capita, which allows us to examine the HBS effect using a long span of data. We therefore measure the non-constant equilibrium of the real pound-dollar rate caused by the HBS effect as follows: $\overline{q}_t = a + b(y_{us_t} - y_{uk_t}), \ \overline{q}_t = a + bt + ct^2 \text{ and } \overline{q}_t = a + bt, \text{ where } y_{us} \text{ and } y_{uk} \text{ are } t$

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real GDP per capita for the USA and UK, respectively. After simple manipulation, equation (1) can be re-written as follows:

$$\Delta \tilde{q}_{t} = \begin{cases} \kappa \rho_{1} + \rho_{1} \tilde{q}_{t-1} + \sum_{i=1}^{m-1} \delta_{1i} \Delta \tilde{q}_{t-i} + \varepsilon_{t} & \text{if } \tilde{q}_{t-d} \leq -\kappa \\ + \rho_{2} \tilde{q}_{t-1} + \sum_{i=1}^{m-1} \delta_{2i} \Delta \tilde{q}_{t-i} + \varepsilon_{t} & \text{if } |\tilde{q}_{t-d}| < \kappa \\ -\kappa \rho_{1} + \rho_{1} \tilde{q}_{t-1} + \sum_{i=1}^{m-1} \delta_{1i} \Delta \tilde{q}_{t-i} + \varepsilon_{t} & \text{if } \tilde{q}_{t-d} \geq \kappa \end{cases}$$

$$(2)$$

where $\tilde{q}_t = q_t - \bar{q}_t$, $\rho_1 = \sum_{i=1}^m \alpha_i - 1$, $\rho_2 = \sum_{i=1}^m \beta_i - 1$, $\delta_{1i} = -(\alpha_{i+1} + ... + \alpha_m)$ and $\delta_{2i} = -(\beta_{i+1} + ... + \beta_m)$, i = 1, 2, ..., m - 1.

Let $\varphi_j = (\rho_j, \delta_{j1}, \ldots, \delta_{j(m-1)})'$, for j = 1, 2; $I_{tL} = I\{\tilde{q}_{t-d} \leq -\kappa\}$, $I_{tM} = I\{|\tilde{q}_{t-d}| \leq \kappa\}$, $I_{tU} = I\{\tilde{q}_{t-d} \geq \kappa\}$; and $x_{t-1} = (\tilde{q}_{t-1}, \Delta \tilde{q}_{t-1}, \ldots, \Delta \tilde{q}_{t-m+1})'$. $I\{\cdot\}$ denotes an indicator variable which takes the value one when the inequality in braces is satisfied, and zero otherwise. With these notations, the model in (2) can be written in a succinct form.

$$\Delta \tilde{q}_t = y_t' \phi + \varepsilon_t \tag{3}$$

where $\phi = (\kappa \rho_{\rm l}, \phi_{\rm l}', \phi_{\rm 2}')'$ and $y_t = (I_{t\rm L} - I_{t\rm U}, (I_{t\rm L} + I_{t\rm U})x_{t-{\rm l}}', I_{t\rm M}x_{t-{\rm l}}')'$.

We adopt a two-step method to estimate equation (3). That is, we regress q_t on deterministic trends and output differentials, respectively, in the first step and then apply the resulting residuals to estimate equation (3). Econometrically, the ordinary least squares (OLS) estimates of slope coefficients in the second step are consistent if the estimates in the first step are consistent. However, the OLS standard errors of the second-step estimates are not robust to possible heteroscedasticity and autocorrelation in residuals. Therefore, conventional test statistics of the related hypothesis in the second step do not have standard distributions. There are two ways to solve for the above problems. One is to estimate the standard errors with White's heteroscedasticity and autocorrelation consistent estimator and the other one is to simulate the limiting distribution of the test statistic through bootstrapping. The latter method is adopted in our paper.

In the case where the HBS effect is measured by deterministic trends, OLS estimates in both the first and second steps are consistent and the standard errors of the second-step estimates are correct. This is because both the right- and left-hand side variables of equation (2) are detrended variables.² However, in the case where the HBS effect is approximated by GDP per capita difference, OLS estimates in the first step are not consistent because of the problem of endogeneity. In this case we estimate the first-step slope coefficient by the method of instrumental variables, and the instrumental variable is chosen as $y_{us_{t+1}} - y_{uk_{t+1}}$.³

²This is implied by the Frisch–Waugh–Lovell theorem which can be found in Green (2008).

³It is worth noting that $y_{us_{t-k}} - y_{uk_{t-k}}$ for $k \ge 1$ is not a valid instrumental variable. The reason comes from the following simplified model: $q_t = a + bx_t + u_t$ and $u_t = \rho u_{t-1} + \varepsilon_t$. The above mode is a special case of the model in (2) with m = 1 and $\kappa = 0$. Assuming that $x_t = y_{us_t} - y_{uk_t}$

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To examine the stationarity of \tilde{q}_i , we test the null hypothesis of a unit root, i.e. $H_0^A: \rho_1 = \rho_2 = 0$, using a supLR statistic proposed by Bec *et al.* (2004). The rejection of H_0^A implies that \tilde{q}_i is a stationary process and can be estimated by the TAR specification without causing econometric problems.

Let $\hat{\phi}$ be the least squares estimator of ϕ in the unrestricted model in (3) under certain values of (κ, d) , $\hat{\varepsilon}_t(\kappa, d) = \Delta \tilde{q}_t - y'_t \hat{\phi}$ and $\hat{\sigma}^2(\kappa, d) = \sum_{t=1}^N \hat{\varepsilon}_t^2(\kappa, d)/N$ (*N* is the sample size). Let ϕ be the estimator of ϕ under the hypothesis of $\rho_1 = \rho_2 = 0$, $\tilde{\varepsilon}_t(\kappa, d) = \Delta \tilde{q}_t - y'_t \tilde{\phi}$ and $\hat{\sigma}^2(\kappa, d) = \sum_{t=1}^N \hat{\varepsilon}_t^2(\kappa, d)/N$. We then construct the likelihood ratio statistic:

$$LR(\kappa, d) = N \frac{\tilde{\sigma}^2(\kappa, d) - \hat{\sigma}^2(\kappa, d)}{\sigma^2(\kappa, d)}$$
(4)

Bec *et al.* (2004) suggest using the supLR statistic for the tests of stationarity, which can be written as

$$\sup_{\substack{\kappa \in [\kappa, \kappa] \\ d = 1, \dots, m}} \operatorname{LR}(\kappa, d)$$
(5)

The estimation of κ and d can be undertaken jointly with the non-linear least squares estimation of the AR parameters by a grid search over (κ , d) in order to maximize the likelihood ratio statistic. The inference of the supLR test is performed by a bootstrap procedure suggested by Hansen (1997, 1999) to simulate the marginal significance levels of the supLR statistic. A detailed description of the bootstrap is given in Appendix A.

It is also interesting to examine the appropriateness of our non-linear band-TAR specification. The following hypotheses are tested sequentially for the above purpose:

$$H_0^{B}: \kappa = 0$$

 $H_0^{C(1)}: \rho_2 = 0$
 $H_0^{C(2)}: \rho_1 = 0$ given $\rho_2 = 0$

It is worth noting that the model in (2) degenerates to a linear model if the hypothesis of $\kappa = 0$ holds. A three-regime symmetric band-TAR specification is appropriate if both hypotheses, H_0^A and H_0^B , are rejected.

The $H_0^{C(1)}$ hypothesis examines whether the deviations from equilibrium real exchange rates follow a specific symmetric band-TAR model with a unit root in the middle regime. If $H_0^{C(1)}$ is not rejected, we impose the restriction of $\rho_2 = 0$ in the model in (2) and then test the hypothesis of $\rho_1 = 0$. If $H_0^{C(2)}$ is rejected, then we claim that the process of deviations from the

and $E(x_tu_l) \neq 0$, then we can derive that $E(x_{t-k}u_t) = \rho^k E(x_{t-k}u_{t-k}) + \rho^{k-1} E(x_{t-k}\varepsilon_{t-k-1}) + \dots + \rho E(x_{t-k}\varepsilon_{t-k-1}) + E(x_{t-k}\varepsilon_{t-k-1}) + E(x_{t-k}\varepsilon_{t-k-1}) + \dots + \rho E(x_{t-k}\varepsilon_{t-k-1}) + E(x_{t-k}\varepsilon_{t-k-1}) + \dots + \rho E(x_{t-k-k-1}) + \dots + \rho E(x_{t$

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equilibrium real exchange rate is I(1) within the no-arbitrage band and is I(0) outside the band.

Equation (3) is estimated by non-linear least squares. Under the assumption of normally distributed errors ε_i , the least squares estimators are equivalent to maximum likelihood estimators. We first examine the unit-root hypothesis by testing the null hypothesis of $H_0^A: \rho_1 = \rho_2 = 0$. The delay parameter, d, and the threshold value, κ , are jointly estimated in testing the hypothesis of H_0^A . We then fix d to the above estimated value (\hat{d}) and re-estimate κ in testing the hypothesis of H_0^B , $H_0^{C(1)}$ and $H_0^{C(2)}$, respectively. Following Peel and Taylor (2002), a likelihood ratio statistic, $LR(\kappa) = N [\tilde{\sigma}^2(\kappa)/\hat{\sigma}^2(\kappa)-1]$, is applied to examine the hypothesis of H_0^B , $H_0^{C(1)}$ and $H_0^{C(2)}$, respectively, where $\tilde{\sigma}^2$ and $\hat{\sigma}^2$ are the restricted and unrestricted residual variances, respectively. A bootstrap procedure is constructed to simulate the marginal significance levels of the LR statistic. A detailed description of the bootstrap procedure is given in Appendix B.

2.2 GIRF Analysis

To further investigate the persistence of short-run deviations of real exchange rates, \tilde{q}_i , we construct the half-life of \tilde{q}_i , which is a summary measure of persistence. The half-life indicates how long it takes for the impact of a unit shock on \tilde{q}_t to dissipate by half. Conventionally, researchers apply an impulse response function to investigate the impacts of shocks on the entire time path of forecast variables (Cheung and Lai, 2000). However, impulse responses constructed from non-linear models are (i) history dependent, (ii) size and sign of current shock dependent, and (iii) future shocks dependent. In other words, shocks of different magnitude and positive and negative shocks of the same magnitude appear to have different dynamic impacts on the entire path of the real exchange rate. A GIRF, provided by Koop et al. (1996), is defined as the effect of a one-time shock on the forecast variables in non-linear models, which successfully overcomes the challenges that arise in defining impulse responses in non-linear models. Following Koop et al. (1996), the GIRF is defined as the difference of two conditional expectations:

$$GIRF_{q}(h, v, \omega_{t-1}) = E[q_{t+h} | v_{t} = v, \omega_{t-1}] - E[q_{t+h} | v_{t} = 0, \omega_{t-1}]$$

where *h* is the forecasting horizon, v_t is the shock to the process at time *t*, ω_{t-1} is the history of the variable and $E[\cdot]$ is the conditional expectation operator. A detailed description of constructing a GIRF is given in Appendix C.

3 Empirical Investigation

Data for the nominal pound-dollar rate and producer price indices over 1885–1994 are obtained from Engel and Kim (1999) and are extended to 2003

$\Delta ilde{q}_t =$	$\begin{cases} \kappa \rho_{1} + \rho_{1} \tilde{q}_{t-1} + \delta_{1} \Delta \tilde{q}_{t-1} + \varepsilon_{t} \\ + \rho_{2} \tilde{q}_{t-1} + \delta_{2} \Delta \tilde{q}_{t-1} + \varepsilon_{t} \\ -\kappa \rho_{1} + \rho_{1} \tilde{q}_{t-1} + \delta_{1} \Delta \tilde{q}_{t-1} + \varepsilon_{t} \end{cases}$		$ \begin{array}{l} \text{if } \tilde{q}_{t-1} \leq -\kappa \\ \text{if } \tilde{q}_{t-1} < \kappa \\ \text{if } \tilde{q}_{t-1} \geq \kappa \end{array} $	
	\overline{q}_t^1	\overline{q}_t^2	\overline{q}_t^3	\overline{q}_t^4
$\mathbf{H}_{\mathrm{o}}^{\mathrm{A}}:\boldsymbol{\rho}_{\mathrm{l}}=\boldsymbol{\rho}_{\mathrm{2}}=0$	15.985* [0.048]	33.087* [0.005]	13.385 [0.231]	12.143 [0.123]
$H_o^B: \kappa = 0$	13.426* [0.016]	12.132** [0.051]		
$H_{o}^{C(1)}: \rho_{2} = 0$	0.084 [0.879]	3.599 [0.529]		_

TABLE 1 RESULTS FOR UNIT-ROOT TESTS

Notes: $\overline{q}_i^1 = a_0 + a_1(y_{us_i} - y_{uk_i})$, $\overline{q}_i^2 = a_0 + a_1t + a_2t^2$, $\overline{q}_i^3 = a_0 + a_1t$ and $\overline{q}_i^4 = a_0$. Figures in square brackets represent the marginal significance levels generated by the bootstrapping method described in the text. * and ** indicate significance at the 5 per cent and 10 per cent levels, respectively.

- indicates that the statistic is not computed.

using the International Monetary Fund's *International Financial Statistics*.⁴ Data for real per capita GDP for the USA and UK are obtained from Global Financial Data.

We consider modeling the real pound-dollar rate by a stationary symmetric band-TAR model with a non-constant equilibrium, in which a unitroot process is likely in the middle regime. This specification is consistent with PPP in the presence of the HBS effect and transaction costs. We first apply the supLR test provided by Bec et al. (2004) to examine the unit-root hypothesis versus a symmetric band-TAR alternative. In the case of a non-constant equilibrium, we assume that it is measured by real GDP per capita difference and deterministic trends, respectively. Following Enders and Granger (1998), we adopt a two-step method in our empirical investigation. We regress the real pound-dollar rate on deterministic trends and the differentials of real GDP per capita, respectively, in the first step and then apply the resulting residuals for further analysis in the second step. The supLR statistic is applied to examine the stationarity of real exchange rates after removing their nonconstant equilibrium. These results are reported in Table 1. A band-TAR(2) model is applied in our empirical analysis.⁵ To ensure there are observations in the middle regime, the grid search covers the 10th to 30th and 70th to 90th percentiles of the arranged sample for the threshold value.

⁴We extend Engel-Kim's data from 1994 to 2003 using the data from *International Financial Statistics*. The base year for the domestic and foreign prices is 1995.

⁵To determine the lag length, we start from a linear AR(1) and then apply the Ljung–Box Q test to check the whiteness of the estimated residuals. If the residuals are non-white, we then increase the lag order by one until they are whitened. The lag order is hence set at 2.

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Results from Table 1 indicate that the specification of the real exchange rate equilibrium is crucial to the TAR stationarity of real exchange rates. Results from the second column of Table 1 indicate that the unit-root hypothesis is rejected at the 5 per cent level of significance if one approximates the HBS effect with real GDP per capita difference. There are several papers that approximate the HBS effect with a linear trend (Cuddington and Liang, 2000; Paya and Peel, 2003; Paya et al., 2003; Peel and Venetis, 2003). It is therefore interesting to examine the appropriateness of this approximation. Results from columns three to five of Table 1 are worth noting since they point out that the unit-root hypothesis is not rejected at conventional levels when the HBS effect is measured by a constant or a linear trend. However, the unit-root hypothesis is rejected if the HBS effect is approximated by a quadratic trend. These results reveal the significance of a nonconstant equilibrium in the long-span real pound-dollar rate, and are consistent with those of Lothian and Taylor (2008). Our results also support the appropriateness of a non-linear trend in proximity to the HBS effect which echo the assertion of Lothian and Taylor (2000, 2008).

To further confirm the appropriate specification of a symmetric band-TAR model, we test the hypothesis of $\kappa = 0$. Results from the third row of Table 1 indicate that the hypothesis of $\kappa = 0$ is rejected at the 10 per cent level of significance, which confirms the appropriate specification for a symmetric band-TAR model in our analysis. We then examine whether the real pounddollar rate follows a unit-root process in the middle regime and a stationary process in the outer regime. We first test the hypothesis of $H_0^{C(l)}$, and find that it is not rejected as shown in Table 1. Since the hypothesis of a unit root in the middle regime ($\rho_2 = 0$) is not rejected, we impose the hypothesis in the model and then test the $H_0^{C(2)}$ hypothesis. We focus on the case where the HBS effect is measured by output differentials and quadratic trends, respectively. Results from Table 2 point out that the $H_0^{C(2)}$ hypothesis is rejected at the 5 per cent level of significance regardless of the measure of HBS effect. Engel and Kim (1999) find evidence of a permanent component in the real pound-dollar rate over the period 1885-1994. However, we find evidence supporting the non-linear mean reversion of the real rate over a similar period after controlling for the HBS effect. Moreover, the specification of the symmetric band-TAR model applied in our empirical analysis is supported empirically.

The upper panel of Table 2 indicates that 25 per cent (28 per cent) of the samples are in the upper (lower) regime and hence 47 per cent of the samples are in the middle regime when the HBS effect is measured by output differentials. As for diagnostic checks, both the Q and Q^2 statistics indicate that there is no serial correlation in residuals and squared residuals. The above results are not sensitive to the measure of the HBS effect as one can find from the bottom panel of Table 2. Results from both Tables 1 and 2 point out that the real pound–dollar rate over 1885–2003 can be modeled by a symmetric

	$\Delta \tilde{q}_{t} = \begin{cases} \kappa \rho_{1} + \rho_{1} \\ \delta_{21} \Delta \tilde{q} \\ -\kappa \rho_{1} - \kappa \rho_{1} \end{cases}$	$\rho_1 \tilde{q}_{t-1} + \delta_{11} \Delta \tilde{q}_{t-1}$ $\tilde{q}_{t-1} + \varepsilon_t$ $+ \rho_1 \tilde{q}_{t-1} + \delta_{11} \Delta \tilde{q}_{t-1}$	$+ \varepsilon_t \qquad \text{if } \tilde{q}_{t-1} \\ \text{if } \tilde{q}_{t-1} \\ + \varepsilon_t \qquad \text{if } \tilde{q}_{t-1} \\ \end{array}$	$\leq -\kappa$ $ <\kappa$ $\geq \kappa$		
]	L	М	τ	J		T 0/
ρ_1	δ_{11}	δ ₂₁	ρ_1	δ_{11}	κ	(L, M, U)
-0.344 (0.19)	0.171 (0.20)	0.239 (0.17) $H_0^{C(2)}: \rho_1 = 0$ Q(6) = 3 $Q^2(6) = 2$	$\tilde{q}_{t} = q_{t} - \bar{q}_{t}^{1}$ -0.344 (0.23) $ \rho_{2} = 0$.96 [0.68] <i>Q</i> 2.57 [0.86] <i>Q</i> ²	0.171 (0.17) LR = 14.104 (12) = 11.78 [(12) = 5.22 [0	0.099 * [0.020] 0.46] .95]	(25, 47, 28)
-0.55 (0.21)	0.34 (0.20)	0.20 (0.16) $H_0^{(2)}: \rho_1 = 0$ Q(6) = 5 $Q^2(6) = 6$	$ \begin{aligned} \tilde{q}_t &= q_t - \bar{q}_t^2 \\ &-0.55 \\ (0.30) \end{aligned} \\ \rho_2 &= 0 \\ .17 & [0.52] Q \\ .92 & [0.33] Q^2 \end{aligned} $	0.34 (0.17) LR = 29.666 (12) = 10.44 [(12) = 13.14 [0.064 * [0.001] 0.58] 0.36]	(26, 47, 27)

 TABLE 2

 Estimation Results for the Band-TAR Model (Annual Data)

band-TAR model with a unit root in the middle regime and the equilibrium band shifts over time to reflect the HBS effect. Bergin *et al.* (2006) apply a model with a continuum of goods differentiated by productivity, monopolistic competition, transaction costs and endogenous tradability to examine the stylized fact of the HBS effect.⁶ They point out that 'the HBS effect has not always been a fact of economic life, and appears to be a phenomenon of only the postwar period'.⁷ Our results are in contrast to their findings since we support the HBS effect based on the data over 1885–2003. This indicates that the above contention made by Bergin *et al.* (2006) is affected if a non-linear model is adopted.

To further investigate the mean-reverting property of the real exchange rate, we construct the half-life of real exchange rates through a GIRF. Rogoff

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Notes: L, M and U represent the regimes which are defined as $\tilde{q}_{t-1} \leq -\kappa$, $|\tilde{q}_{t-1}| < \kappa$ and $\tilde{q}_{t-1} \geq \kappa$, respectively. Figures in parentheses denote the estimated standard errors and those within square brackets represent the marginal significance levels. Q(j) and $Q^2(j)$ denote the Ljung–Box autocorrelation test statistics for residuals and squared residuals, respectively, for up to *j*th-order autocorrelations, which have a chi-squared distribution with *j* degrees of freedom. *indicates significance at the 5 per cent level. T% represents the percentage of observations in the L, M and U regimes, respectively. Same as notes in Table 1.

⁶Let us assume that the pre-condition of the HBS effect is that technological shocks hit traded sectors. Suppose shocks hit non-traded sectors initially; then those that receive positive technology shocks and pay for transaction costs become traded sectors, which meets the pre-condition of the HBS effect and it arises endogenously.

⁷Their empirical evidence reveals that the effect virtually vanishes from the data if one looks back 50 years or more.

(1996) points out that the half-life of the real exchange rate from a linear model is about three to five years, which is too long to be consistent with the explanation of nominal rigidities. With data spanning over two centuries, Lothian and Taylor (1996) find that the half-life for the pound–dollar rate is about six years. It is worth noting that the equilibrium band shifts over time and hence deviations from the non-constant equilibrium of real exchange rates (\tilde{q}_i) are applied in constructing the GIRF. We examine how long it takes for \tilde{q}_i , after facing a unit of shock, to dissipate by half relative to its equilibrium band edge.

The plots of the GIRF under different sizes of shock based on the symmetric band-TAR model are given in Figs 1 and 2 with the respective output differential and trend measure of the HBS effect. The horizontal line is the estimated threshold value. It is worth noting that, in a band-TAR model, the entire interval $[-\kappa, \kappa]$ of the band is the equilibrium of the real exchange rate in which there is no arbitrage. We are interested in how long it takes for short-run deviations to revert back to the equilibrium band. Therefore, the half-life should be constructed relative to the band edge rather than the center of the equilibrium. Findings from columns 4 and 5 of Table 3 indicate that the calculated half-life is two years with larger shocks when the HBS effect is measured by quadratic trends as one can see from columns 2 and 3 of Table 3. Modeling the non-linearities of real exchange rates with an ESTAR model, Lothian and Taylor (2008) find that the half-life of real exchange rates is one year or less with larger shocks. Our



FIG. 1 GIRF for Different Sizes of Shock (Annual Data, $\tilde{q}_t = q_t - \overline{q}_t^1$) *Note:* Shocks of 0.1, 1.0, 1.5 and 2.0 standard deviations are plotted from bottom to top.



FIG. 2 GIRF for Different Sizes of Shock (Annual Data, $\tilde{q}_t = q_t - \bar{q}_t^2$) *Note:* Shocks of 0.1, 1.0, 1.5 and 2.0 standard deviations are plotted from bottom to top.

TABLE 3					
ESTIMATED	HALF-LIFE	(IN	YEARS)		

	$\tilde{q}_t = q_t - \overline{q}_t^1$		$\tilde{q}_t = q_t - \overline{q}_t^2$		
	Sizes of the shock (s.e.)	h	Sizes of the shock (s.e.)	h	
Within the band	0.1		0.1		
	1.0		1.0		
Outside of the band	1.5	2	1.5	2	
	2.0	2	2.0	2	

Notes: h represents the half-life calculated relative to the band edge. s.e. indicates standard deviations. Same as notes in Table 1.

results point out that the real exchange rate reverts to its equilibrium at a rate much faster than the 'glacial rates' previously reported for linear models. These results are also consistent with those of Lothian and Taylor (2008). In sum, we conclude that our non-linear modeling strategy, after controlling for the HBS effect, provides a solution to the PPP puzzle outlined in Rogoff (1996).

4 CONCLUSION

The purpose of this paper is to apply a symmetric band-TAR model to investigate the non-linear adjustment of the real pound–dollar rate during the period 1885–2003. We find that the unit-root hypothesis of real exchange rates is rejected at the 5 per cent level of significance if one approximates the

non-constant equilibrium with either the differential of real GDP per capita or a non-linear trend. After removing the non-constant equilibrium of the real pound–dollar rate, we find that it is appropriate to model the real rate in a symmetric band-TAR model, in which it exhibits the property of non-linear mean reversion. Moreover, the generalized impulse response analysis points out that the rate of adjustment for larger shocks is much faster than the 'glacial rates' previously reported for linear models.

Appendix A

Simulations for the Marginal Significance Level of the supLR Statistic

Here we give a detailed description regarding how to obtain the marginal significance level of the supLR statistic for testing the null hypothesis of $H_o^A: \rho_1 = \rho_2 = 0$. The bootstrap procedure suggested by Hansen (1997, 1999) is described as follows.

- 1. Estimate equation (3) under the null hypothesis H_0^A , i.e. $\Delta \tilde{q}_t = (\Sigma_{i=1}^{m-1} \delta_{1i} \Delta \tilde{q}_{t-i}) I\{\tilde{q}_{t-d} \le -\hat{\kappa} \text{ or } \tilde{q}_{t-d} \ge \hat{\kappa}\} + (\Sigma_{i=1}^{m-1} \delta_{2i} \Delta \tilde{q}_{t-i}) I\{|\tilde{q}_{t-d}| < \hat{\kappa}\} + \varepsilon_t$ by OLS, where $\hat{\kappa}$ is taken from the estimation of the unrestricted model in (3). We obtain the estimated parameters $\hat{\delta}_{1i}$, $\hat{\delta}_{2i}$ and residuals $\hat{\varepsilon}_t$.
- 2. Generate a random sample $\varepsilon_t^*, t = 1, ..., T$, by sampling with replacement from the residuals obtained from the previous step. Next, recursively generate a sample \tilde{q}_t^* with the estimated parameters of the model under the null hypothesis,

$$\tilde{q}_{i}^{*} = \tilde{q}_{i-1}^{*} + (\Sigma_{i=1}^{m-1}\hat{\delta}_{1i}\Delta\tilde{q}_{i-i}^{*})I\{\tilde{q}_{i-d}^{*} \le -\hat{\kappa} \text{ or } \tilde{q}_{i-d}^{*} \ge \hat{\kappa}\} + (\Sigma_{i=1}^{m-1}\hat{\delta}_{2i}\Delta\tilde{q}_{i-i}^{*})I\{|\tilde{q}_{i-d}^{*}| < \hat{\kappa}\} + \mathcal{E}_{i}^{*}$$

The initial conditions are given by the first *m* observations, $\tilde{q}_0, \tilde{q}_{-1}, \ldots, \tilde{q}_{-m+1}$.

- 3. Form the supLR* statistic, defined in equation (5), by estimating the unrestricted model and the restricted model, i.e. the model in step 1, by using the generated series \tilde{q}_t^* .
- 4. Repeat steps 2 to 4 1000 times and obtain the empirical distribution of the supLR* statistic. The marginal significance level is the percentage of simulated supLR* statistics which exceed the actual statistic.

APPENDIX B

Simulations for the Marginal Significance Level of the LR Statistic

The bootstrap procedure to obtain the marginal significance level of the LR statistic is suggested by Hansen (1997, 1999) and is described as follows.

1. Estimate equation (3) under the null hypothesis $\mathbf{H}_{o}^{B}: \kappa = 0$, i.e. $\Delta \tilde{q}_{i} = \rho \tilde{q}_{i-1} + \sum_{i=1}^{m-1} \delta_{i} \Delta \tilde{q}_{i-i} + \varepsilon_{i}$ by OLS. We obtain the estimated parameters $\hat{\rho}$, $\hat{\delta}_{i}$ and residuals $\hat{\varepsilon}_{i}$. 2. Generate a random sample ε_t^* , t = 1, ..., T, by sampling with replacement from the residuals obtained from the previous step. Next, recursively generate a sample \tilde{q}_t^* with the estimated parameters of the model under the null hypothesis:

$$\tilde{q}_t^* = (\hat{\rho} + 1)\tilde{q}_{t-1}^* + \sum_{i=1}^{m-1} \hat{\delta}_i \Delta \tilde{q}_{t-i}^* + \varepsilon_t^*$$

The initial conditions are given by the first *m* observations, $\tilde{q}_0, \tilde{q}_{-1}, ..., \tilde{q}_{-m+1}$.

- 3. Form the LR* statistic by estimating the unrestricted model and the restricted model, i.e. the model in step 1, by using the generated series \tilde{q}_i^* .
- 4. Repeat steps 2 to 4 1000 times and obtain the empirical distribution of the LR* statistic. The marginal significance level is the percentage of simulated LR* statistics which exceed the actual statistic.

Appendix C

Simulations for the GIRF

The purpose of this appendix is to provide a detailed description of constructing the generalized impulse function, which is defined as $E[q_{t+h}|v_t = v, \omega_{t-1}] - E[q_{t+h}|v_t = 0, \omega_{t-1}]$. The model that the variable \tilde{q}_{t+h} follows is

$$\Delta \tilde{q}_{t} = \begin{cases} \kappa \rho_{1} + \rho_{1} \tilde{q}_{t-1} + \delta_{11} \Delta \tilde{q}_{t-1} + \varepsilon_{t} & \text{if } \tilde{q}_{t-1} \leq -\kappa \\ \delta_{21} \Delta \tilde{q}_{t-1} + \varepsilon_{t} & \text{if } |\tilde{q}_{t-1}| < \kappa \\ -\kappa \rho_{1} + \rho_{1} \tilde{q}_{t-1} + \delta_{11} \Delta \tilde{q}_{t-1} + \varepsilon_{t} & \text{if } \tilde{q}_{t-1} \geq \kappa \end{cases}$$
(A1)

The detailed simulation procedures, suggested by Koop *et al.* (1996), are given as follows.

- 1. Let $\omega_{t-1} = \{\tilde{q}_{t-1}, \dots, \tilde{q}_{t-k}\}$ be the given history of trend-adjusted real exchange rates, \tilde{q}_t . We then estimate equation (A1) to obtain parameter estimates ($\hat{\kappa}$, $\hat{\rho}_1$, $\hat{\delta}_{11}$, $\hat{\delta}_{21}$) and estimated residuals ($\hat{\varepsilon}_t$). The parameter estimates are reported in Table 2.
- 2. Generate a random sample v_{t+h}^0 , h = 0, ..., H, by sampling with replacement from the residuals obtained from step 1. Given v_{t+h}^0 , h = 0, ..., H and the parameter estimates from step 1, we then calculate the realizations of $\tilde{q}_{t+h}^{*(1)}(v_t = 0, \omega_{t-1})$ for h = 0, 1, ..., H by iterating the following non-linear model given the initial condition ω_{t-1} .

$$\tilde{q}_{l+h}^{*} = \begin{cases} \tilde{q}_{l+h-1}^{*} + \hat{\kappa} \hat{\rho}_{1} + \hat{\rho}_{1} \tilde{q}_{l+h-1}^{*} + \hat{\delta}_{11} (\tilde{q}_{l+h-1}^{*} - \tilde{q}_{l+h-2}^{*}) + \mathbf{v}_{l+h}^{0} & \text{if } \tilde{q}_{l+h-1}^{*} \leq -\hat{\kappa} \\ \tilde{q}_{l+h-1}^{*} + \hat{\delta}_{21} (\tilde{q}_{l+h-1}^{*} - \tilde{q}_{l+h-2}^{*}) + \mathbf{v}_{l+h}^{0} & \text{if } |\tilde{q}_{l+h-1}^{*}| < \hat{\kappa} \\ \tilde{q}_{l+h-1}^{*} - \hat{\kappa} \hat{\rho}_{1} + \hat{\rho}_{1} \tilde{q}_{l+h-1}^{*} + \hat{\delta}_{11} (\tilde{q}_{l+h-1}^{*} - \tilde{q}_{l+h-2}^{*}) + \mathbf{v}_{l+h}^{0} & \text{if } \tilde{q}_{l+h-1}^{*} \geq \hat{\kappa} \end{cases}$$
(A2)

for h = 0, 1, ..., H.

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3. Generate a random sample v_{t+h}^v , h = 0, ..., H, where v_t^v is set to be the 0.1 or 1.0 or 1.5 or 2.0 standard deviations of the estimated residuals from step 1 and v_{t+h}^v , h = 1, ..., H, are generated by resampling with replacement from the residuals obtained from step 1. Given the parameter estimates in step 1 and v_{t+h}^v , h = 1, ..., h = 1, ...,

0, ..., *H*, we then calculate the realizations of $\tilde{q}_{t+h}^{*(1)}(v_t = v, \omega_{t-1})$ for h = 0, 1, ..., H by iterating equation (A2) given the initial condition ω_{t-1} in which v_{t+h}^0 is replaced by v_{t+h}^v .

- 4. Repeat steps 3 and 4 *N* times (*N* = 20,000) and obtain $\tilde{q}_{t+h}^{*(n)}(v_t = v, \omega_{t-1})$ and $\tilde{q}_{t+h}^{*(n)}(v_t = 0, \omega_{t-1})$ for n = 1, ..., N. As $N \to \infty$, by the law of large numbers the realizations of $E[\tilde{q}_{t+h}^{*}|v_t = v, \omega_{t-1}]$ and $E[\tilde{q}_{t+h}^{*}|v_t = 0, \omega_{t-1}]$ are approximated by the average across individual replications, $(1/N) \sum_{i=1}^{N} \tilde{q}_{t+h}^{*(i)}(v_t = v, \omega_{t-1})$ and $(1/N) \sum_{i=1}^{N} \tilde{q}_{t+h}^{*(i)}(v_t = 0, \omega_{t-1})$, respectively.
- 5. Take the difference between the two averages to form the estimates of GIRF.

$$GIRF_{q}(h, \nu, \omega_{t-1}) = E[q_{t+h}^{*} | \nu_{t} = \nu, \omega_{t-1}] - E[q_{t+h}^{*} | \nu_{t} = 0, \omega_{t-1}]$$
$$= (1/N) \sum_{i=1}^{N} \tilde{q}_{t+h}^{*(i)}(\nu_{t} = \nu, \omega_{t-1}) - (1/N) \sum_{i=1}^{N} \tilde{q}_{t+h}^{*(i)}(\nu_{t} = 0, \omega_{t-1})$$

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