A Panel Unit-Root Test with Smooth Breaks and Cross-Sectional Dependence

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Abstract

This paper develops a simple panel unit-root test that accommodates cross-sectional dependence among variables and smooth structural changes in deterministic components. The proposed test is the simple average of the individual statistics constructed from the breaks and cross-sectional dependence augmented Dickey-Fuller ($BCADF$) regression. Applying the sequential limit approach, this paper shows that the asymptotic distribution of the $BCADF$ statistic is free of nuisance parameters as N, T go to infinity. We also extend our analysis to the case where shocks are serially correlated. The limiting distribution of the average $BCADF$ statistic is shown to exist and its critical values are tabulated. Monte-Carlo experiments point out that the size and power of the average $BCADF$ statistic are generally good as long as T is greater than fifty. The test is then applied to examine the validity of long-run purchasing power parity.

Keywords: Panel unit-root test, Fourier approximation, Cross-sectional dependence, Purchasing power parity.

JEL classification: C12, C33.

1 Introduction

The development of panel unit-root tests had been a hot research topic during the past decade. The first generation panel unit-root tests either neglect cross-sectional dependence or deal with cross dependence by bootstrapping critical values but fail to derive the asymptotic distribution of test statistics (Levin et al., 2002; Im et al., 2003, IPS; Maddala and Wu, 1999). However, controlling contemporaneous correlation across individuals is important to a panel unit-root test since leaving out cross-sectional dependencies would lead to serious size distortions and power loss (O'Connell, 1998). The second generation models allow for cross-sectional dependence and derive the asymptotic distribution of test statistics (Chang, 2002; Choi, 2002; Phillips and Sul, 2003; Bai and Ng, 2004; Moon and Perron, 2004; Smith et al., 2004; Breitung and Das, 2005; Choi and Chue, 2007; Pesaran, 2007; Pesaran et al., 2009). The above-mentioned articles assume away structural changes in their models.

Several articles allow panel unit-root tests with structural breaks (Im et al., 2005; Carrion-i-Silvestre et al., 2001; Carrion-i-Silvestre et al., 2005). These tests, however, assume that the variables in the panel are cross-sectionally independent. Recently, Bai and Carrion-i-Silvestre (2009) and Im et al. (2010, ILT) have proposed panel unit-root tests in the presence of multiple structural changes and cross-sectional dependence. Bai and Carrion-i-Silvestre (2009) propose a modified Sargan-Bhargava (1983, MSB) test in the panel setting. Although this test is invariant to both mean and trend break parameters, the limiting distribution of the individual MSB ($MSB^*(\lambda)$) test depends on the number of structural breaks. Following the cross-sectionally augmented procedure of Pesaran (2007), Im et al. (2010) develop an LM-type panel unit-root test to account for possible heterogeneity in both the level and the trend of the series. The ILT test is invariant to the nuisance parameters but its limiting distribution depends on the number of trend breaks.

To take into account the smooth breaks in the deterministic components of a series, several articles develop unit-root tests based on Gallant's (1981) Flexible Fourier Form. It has been observed that a Fourier approximation to deterministic components can, to any desired degree of accuracy, capture the behavior of a deterministic function of unknown form even if the function itself is not periodic (Gallant, 1981; Davies, 1987; Bierens, 1997; Becker et al., 2004; Enders and Lee, 2011). Moreover, Enders and Lee (2011) point out that a single-frequency-component Fourier function works better than dummy variable methods regardless of whether the breaks are instantaneous or smooth. Another advantage of the Fourier approximation is that it involves only the determination of the appropriate frequency component in the model and hence avoids the complication of selecting break dates, the number of breaks and the form of breaks. Because of the above interesting results, Enders and Lee (2009) and Rodrigues and Taylor (2009) develop unit-root tests for time series data by augmenting the conventional OLS and GLS regression to include a single-frequency-component Fourier function. They find that their proposed tests are useful since they are robust to a variety of possible break mechanisms in the deterministic trend function of unknown forms and numbers.

To allow for both heterogeneous breaks and cross-sectional correlations, this paper generalizes the cross-sectionally augmented ADF ($CADF$) regression proposed by Pesaran (2007) to incorporate a single-frequency-component Fourier function with heterogeneous amplitudes. The included Fourier function is used to approximate the unknown multiple structural breaks (Becker et al., 2006; Enders and Lee, 20011). We thus construct the individual breaks and cross-sectional dependence augmented ADF ($BCADF$) statistics and their simple average.

The latter statistic is referred to as the breaks and cross-sectional dependence augmented IPS ($BCIPS$) statistic. An important advantage of our proposed tests is their simplicity in empirical applications.

To analyze the impact of Fourier terms in the $BCADF$ regression in both finite and large sample size (T), new asymptotic results of $BCADF$ and $BCIPS$ statistics are derived based on the sequential limit approach wherein the panel size (N) tends to infinity followed by T goes to infinity. In the case of serially uncorrelated errors, Theorem 1 shows that the asymptotic distribution of $BCADF$, under a fixed T, is not free of nuisance parameters even when N tends to infinity. This is because the inclusion of the Fourier terms in the null process makes the stochastic part of the regressors composed of the terms of different orders in probability under a fixed T. Theorem 2 shows that the dependence of the asymptotic distribution on nuisance parameters vanishes as both N and T sequentially tend to infinity. This is due to the fact that stochastic trend terms asymptotically dominate Fourier terms as T tends to infinity. The Corollary examines the limiting distribution of Pesaran's $CADF$ statistic (that assumes no breaks) in the presence of Fourier form breaks. We show that the asymptotic distribution of the $CADF$ statistic under a fixed T includes omitted-variable bias even when N tends to infinity, although the bias vanishes when both N and T approach infinity. The limiting distribution of the truncated version of the $BCIPS$ statistic, $BCIPS^*$, is also shown to exist. Furthermore, this paper extends the discussion to the case where the residuals are serially correlated. By adding a lagged first-differenced variable and its cross-sectional mean in the regression, Theorem 3 shows that the $BCADF$ statistic, under serially correlated errors, has the same asymptotic distribution as that in Theorem 2 when both N, T tend to infinity.

Although the asymptotic distribution of the $BCIPS^*$ statistic exists, it is not analytically tractable, this paper therefore tabulates its critical values under different N, T and k. The size and power of the $BCIPS^*$ statistic under different scenarios are then explored by Monte-Carlo simulations. Simulation results indicate that the size and power of our proposed statistic are generally good regardless of N and k as long as T is greater than fifty. On the contrary, the size distortion of Pesaran's $CIPS$ statistic that ignores the Fourier terms in regression is quite serious in many cases even when T is as large as 400. Finally, the proposed panel unit-root test is applied to investigate the long-run purchasing power parity (PPP) over the post-Bretton Woods period.

The remainder of this paper is organized as follows. Section 2 sets out the basic dynamic heterogeneous panel data model with smooth breaks. The cross-sectional dependence across variables is modeled by an unobservable stationary common factor and the smooth breaks in deterministic terms are approximated by a Fourier function. In Section 3, we derive the null distribution of the individual $BCADF$ statistic with serially uncorrelated errors, discuss the $BCADF$-based panel unit-root test, and extend our results to the case with serially correlated errors. Section 4 examines the finite-sample properties of the proposed panel unit-root test, $BCIPS$, by Monte-Carlo simulations. Section 5 provides an empirical application. Finally, Section 6 concludes.

2 The Model

Let y_{it} be an observation on the ith cross-sectional unit at time t and suppose that it is generated according to the following simple dynamic linear heterogeneous panel data model with an unknown time-dependent deterministic term $\delta_i(t)$:

$$(1-\phi_i L)(y_{it}-\delta_i(t)-\varsigma_i\cdot t)=u_{it},\ \ u_{it}=\gamma_i f_t+\varepsilon_{it},\ \ t=1,..,T,\ \ i=1,..,N, \tag{1}$$

where $\varsigma_i t$ is a linear trend, f_t is an 1×1 unobserved stationary stochastic common factor, γ_i is the associated factor loading reflecting the degree of contemporaneous correlation across individuals and the ε_{it} is an idiosyncratic error and the initial value, y_{i0}, is given. This paper adopts the Fourier function with a single frequency component to approximate $\delta_i(t)$, so that

$$\delta_i(t)\approx\varpi_{i,k,t}=\mu_i+\alpha_{i,1}\sin(2\pi kt/T)+\alpha_{i,2}\cos(2\pi kt/T), \tag{2}$$

where k is the frequency component reflecting the number of cycles in the sample period and it is assumed to be homogeneous across agents. $\alpha_{i,1}$ and $\alpha_{i,2}$ measure the heterogeneous amplitude and displacement of the sinusoidal component of the deterministic term across agents. $\varpi_{i,k,t}$ in (2) captures smooth breaks in deterministic components. Substituting (2) into (1), we obtain[1]

$$\Delta y_{it}=\beta_i(y_{i,t-1}-\boldsymbol{\alpha}_i'\mathbf{d}_{t-1})+\boldsymbol{\alpha}_i'\Delta\mathbf{d}_t+\gamma_i f_t+\varepsilon_{it}, \tag{3}$$

where $\Delta y_{it}=y_{it}-y_{i,t-1}$, $\mathbf{d}_t=(1,\sin(2\pi kt/T),\cos(2\pi kt/T),t)'$ is a 4×1 vector of deter-

[1] A restriction embedded in a Fourier function is that the starting and ending value of the function are the same. The introduction of a time trend, $\varsigma_i t$, in the function removes the above restriction. As such, changes in the intercept and slope of a deterministic function can be captured by the Fourier approximation. Therefore our proposed panel unit-root tests allow for breaks in both the level and trend of the series under investigation.

ministic common effect, $\Delta\mathbf{d}_t = (0, \Delta\sin(2\pi kt/T), \Delta\cos(2\pi kt/T), 1)'$, $\beta_i = -(1-\phi_i)$, and $\boldsymbol{\alpha}_i = (\mu_i, \alpha_{i,1}, \alpha_{i,2}, \varsigma_i)'$. The unit-root hypothesis, $\phi_i = 1$, can be expressed as:

$$H_0 : \beta_i = 0 \quad \forall i, \tag{4}$$

against the possibly heterogeneous alternatives,

$$H_1 : \ \beta_i < 0, \ i = 1, 2, ..., N_1; \ \beta_i = 0, \ i = N_1+1, N_1+2, ..., N. \tag{5}$$

Under the above null hypothesis, equation (3) can be solved for y_{it} as follows:

$$y_{it} = \tilde{y}_{i0} + \boldsymbol{\alpha}_i'\mathbf{d}_t + \gamma_i \mathbf{s}_{ft} + \mathbf{s}_{it},$$

where $\boldsymbol{\alpha}_i'\mathbf{d}_t = \mu_i + \alpha_{i,1}\sin(2\pi kt/T) + \alpha_{i,2}\cos(2\pi kt/T) + \varsigma_i t$, $s_{ft} = f_1 + f_2 + ... + f_t$, $s_{it} = \varepsilon_{i1} + \varepsilon_{i2} + ... + \varepsilon_{it}$, $\tilde{y}_{i0} = y_{i0} - \boldsymbol{\alpha}_i'\mathbf{d}_0$. Therefore, under H_0, y_{it} is composed of a deterministic component with a Fourier element, $\tilde{y}_{i0} + \boldsymbol{\alpha}_i'\mathbf{d}_t$, a common stochastic component, $\mathbf{s}_{ft} \sim I(1)$, and an idiosyncratic component, $s_{it} \sim I(1)$. We do not assume $\alpha_{i,1} = \alpha_{i,2} = 0$ under the null hypothesis. As such, heterogeneous breaks exist under the null and alternative hypothesis in (4) and (5), respectively, and hence our proposed tests avoid the possibility of spuriously rejecting a unit-root hypothesis (Leybourne et al., 1998 and Enders and Lee, 20011). The following assumptions are required for deriving the null distribution of a series-specific unit-root test.

Assumption 1 (Idiosyncratic Errors): The idiosyncratic error, ε_{it}, with a zero mean, a constant variance σ_i^2, and a finite fourth-order moment, is independently distributed across i and t and is independent of f_s for all i, t, s.

Assumption 2 (Common Factors): The common factor, f_t, is serially uncorrelated and has a zero mean, a constant variance σ_f^2, and a finite fourth-order moment. Besides, σ_f^2 is assumed to be unity without loss of generality.

Assumption 3: (Factor Loadings): The factor loading parameter, γ_i, satisfying $\operatorname{plim}\frac{1}{N}\sum_{i=1}^{N}\gamma_i = \gamma^* \neq 0$, is independently distributed across i and is independent of the idiosyncratic ε_{it} and of the common factor f_s for all i, t, s.

Assumption 4: (Fourier Amplitude Coefficients): The Fourier amplitude coefficients, $\alpha_{i,1}$ and $\alpha_{i,2}$, are nonrandom parameters satisfying $\lim \frac{1}{N}\sum_{i=1}^{N}\alpha_{i,1} = \alpha_1^* \neq 0$ and $\lim \frac{1}{N}\sum_{i=1}^{N}\alpha_{i,2} = \alpha_2^* \neq 0$.

Assumptions 1-3 together imply that the composite error, u_{it}, is serially uncorrelated and the case with serially correlated errors will be discussed in Section 3.3.

3 Breaks and Cross Dependence Augmented Unit-Root Tests

Theorems 1-3 and the Corollary in this section derive the asymptotic distribution of the series-specific unit-root test statistic under the null hypothesis in (4). Note that all order results and the proofs of theorems given in the Appendix are derived for the case where $\mathbf{d}_t = (1, \sin(2\pi kt/T), \cos(2\pi kt/T))'$, $t = 1, 2.., T$, which implies $\Delta\mathbf{d}_t = (0, \Delta\sin(2\pi kt/T), \Delta\cos(2\pi kt/T))'$. The asymptotic results for the case where $\mathbf{d}_t = (1, \sin(2\pi kt/T), \cos(2\pi kt/T), t)'$ can be derived in a similar manner.

3.1 Unit-root Tests with Serially Uncorrelated Errors

Although f_t in (3) is unobservable, it can be filtered out by taking the cross-sectional average of (3) (Pesaran, 2006). By doing so, the common factor f_t can be measured by the linear combination of $\Delta\sin(2\pi kt/T)$, $\Delta\cos(2\pi kt/T)$, $\sin(2\pi k(t-1)/T)$, $\cos(2\pi k(t-1)/T)$, $\Delta\overline{y}_t$ and $\overline{y}_{t-1}$, where $\Delta\overline{y}_t = \frac{1}{N}\sum_{i=1}^{N}\Delta y_{it}$ and $\overline{y}_t = \frac{1}{N}\sum_{i=1}^{N} y_{i,t-1}$. We therefore regress the following breaks and cross dependence augmented Dickey-Fuller equation using OLS:[2]

$$\begin{aligned}\Delta y_{it} &= c_{i,0} + c_{i,1}\Delta\sin(2\pi kt/T) + c_{i,2}\Delta\cos(2\pi kt/T) + c_{i,3}\overline{y}_{t-1} \\ &\quad + c_{i,4}\Delta\overline{y}_t + b_i y_{i,t-1} + e_{it}. \end{aligned} \tag{6}$$

The t-statistic of the estimate of b_i ($\hat{b}_i$) is then applied to examine the unit-root hypothesis and can be expressed as

$$t_i(N,T) = \frac{\Delta\mathbf{y}_i'\mathbf{M}_z\mathbf{y}_{i,-1}}{\hat{\sigma}_i(\mathbf{y}_{i,-1}'\mathbf{M}_z\mathbf{y}_{i,-1})^{1/2}}, \tag{7}$$

where $\Delta\mathbf{y}_i = (\Delta y_{i1}, \Delta y_{i2}, ..., \Delta y_{iT})'$, $\mathbf{y}_{i,-1} = (y_{i0}, y_{i1}, ..., y_{i,T-1})'$, $\hat{\sigma}_i^2 = \frac{\Delta\mathbf{y}_i'\mathbf{M}_{i,z}\Delta\mathbf{y}_i}{T-6}$, $\mathbf{M}_z = \mathbf{I}_T - \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$, $\mathbf{M}_{i,z} = \mathbf{I}_T - \mathbf{G}_i(\mathbf{G}_i'\mathbf{G}_i)^{-1}\mathbf{G}_i'$, $\mathbf{G}_i = (\mathbf{Z}, \mathbf{y}_{i,-1})$, $\mathbf{Z} = (\Delta\overline{\mathbf{y}}, \boldsymbol{\tau}, \boldsymbol{\Upsilon}_1, \boldsymbol{\Upsilon}_2, \overline{\mathbf{y}}_{-1})$, $\boldsymbol{\tau} = (1, 1, ..., 1)'$, $\boldsymbol{\Upsilon}_1 = (\Delta\sin(2\pi k1/T), \Delta\sin(2\pi k2/T), ..., \Delta\sin(2\pi kT/T))'$, $\boldsymbol{\Upsilon}_2 = (\Delta\cos(2\pi k1/T),$

[2] Following the argument by Enders and Lee (2009, p.14), we omit $\sin(2\pi k(t-1)/T)$ and $\cos(2\pi k(t-1)/T)$ to avoid collinearity in the regression. See (L5) and (L6) in Lemma 1 in the appendix.

$\Delta\cos(2\pi k2/T), ..., \Delta\cos(2\pi kT/T))'$, $\Delta\overline{\mathbf{y}} = (\Delta\overline{y}_1, \Delta\overline{y}_2, ..., \Delta\overline{y}_T)'$, $\overline{\mathbf{y}}_{-1} = (\overline{y}_0, \overline{y}_1, ..., \overline{y}_{T-1})'$.

It should be noted that the exact null distribution of $t_i(N,T)$ defined in (7) is affected by nuisance parameters through their effect on the matrix $\mathbf{M}_z$ and $\mathbf{M}_{i,z}$. In the model of Pesaran (2007), the effect of the nuisance parameters on $t_i(N,T)$ vanishes as $N \to \infty$. However, the following theorems show that the asymptotic distribution of $t_i(N,T)$ in our model is free of nuisance parameters only when both $N \to \infty$ and $T \to \infty$.

Theorem 1:

Let y_{it} be generated based on (3) under the null hypothesis in (4) with the cross-sectional mean of the initial observation $\overline{y}_0$ being zero. Suppose that Assumptions 1-4 hold and $\overline{\gamma} \neq 0$, then the null distribution of $t_i(N,T)$ given by (7) will not be free of nuisance parameters as $N \to \infty$ for any fixed $T > 6$. In particular, we have

$$t_i(N,T) \xrightarrow{N} \frac{\frac{\boldsymbol{\varepsilon}_i'\mathbf{s}_{i,-1}}{\sigma_i^2 T} - (\mathbf{q}_{iT}'\boldsymbol{\Gamma}_T')(\boldsymbol{\Gamma}_T\boldsymbol{\Psi}_{fT}\boldsymbol{\Gamma}_T')^{-1}(\boldsymbol{\Gamma}_T\mathbf{h}_{iT})}{J_1 \times J_2}, \tag{8}$$

where

$$J_1 = \left(\frac{\boldsymbol{\varepsilon}_i'\boldsymbol{\varepsilon}_i}{\sigma_i^2(T-6)} - \frac{(\mathbf{d}_{iT}'\boldsymbol{\Theta}_T')(\boldsymbol{\Theta}_T\boldsymbol{\Xi}_{iT}\boldsymbol{\Theta}_T')^{-1}(\boldsymbol{\Theta}_T\mathbf{d}_{iT})}{T-6}\right)^{1/2}, \tag{9}$$

$$J_2 = \left(\frac{\mathbf{s}_{i,-1}'\mathbf{s}_{i,-1}}{\sigma_i^2 T^2} - (\mathbf{h}_{iT}'\boldsymbol{\Gamma}_T')(\boldsymbol{\Gamma}_T\boldsymbol{\Psi}_{fT}\boldsymbol{\Gamma}_T')^{-1}(\boldsymbol{\Gamma}_T\mathbf{h}_{iT})\right)^{1/2}, \tag{10}$$

$$\boldsymbol{\Gamma}_T = \begin{bmatrix} \gamma^* & 0 & T^{-1}\alpha_1^* & T^{-1}\alpha_2^* & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & T^{-1/2}\alpha_1^* & T^{-1/2}\alpha_2^* & \gamma^* \end{bmatrix}, \tag{11}$$

$$\boldsymbol{\Xi}_{iT} = \begin{bmatrix} \boldsymbol{\Xi}_{iT}^{(1)} & \boldsymbol{\Xi}_{iT}^{(2)} \end{bmatrix}, \quad \boldsymbol{\Psi}_{fT} = \begin{bmatrix} \boldsymbol{\Psi}_{fT}^{(1)} & \boldsymbol{\Psi}_{fT}^{(2)} \end{bmatrix}, \tag{12}$$

$$\boldsymbol{\Theta}_T = \begin{bmatrix} & & & & & & & 0 \\ & & & & & & & 0 \\ & & & \boldsymbol{\Gamma}_T & & & & 0 \\ & & & & & & & 0 \\ & & & & & & & 0 \\ 0 & T^{-1/2}y_0 & 0 & 0 & T^{-1/2}\alpha_{i,1} & T^{-1/2}\alpha_{i,2} & \gamma_i & 1 \end{bmatrix}, \tag{13}$$

$$
\mathbf{q}_{iT} = \begin{bmatrix} \frac{\mathbf{f}'\boldsymbol{\varepsilon}_i}{\sigma_i\sqrt{T}} \\ \frac{\boldsymbol{\tau}'\boldsymbol{\varepsilon}_i}{\sigma_i\sqrt{T}} \\ \frac{\sqrt{T}(\boldsymbol{\Upsilon}_1'\boldsymbol{\varepsilon}_i)}{\sigma_i} \\ \frac{\sqrt{T}(\boldsymbol{\Upsilon}_2'\boldsymbol{\varepsilon}_i)}{\sigma_i} \\ \frac{\mathbf{s}_{\Upsilon_1,-1}'\boldsymbol{\varepsilon}_i}{\sigma_i\sqrt{T}} \\ \frac{\mathbf{s}_{\Upsilon_2,-1}'\boldsymbol{\varepsilon}_i}{\sigma_i\sqrt{T}} \\ \frac{\mathbf{s}_{f,-1}'\boldsymbol{\varepsilon}_i}{\sigma_i T} \end{bmatrix}, \quad \mathbf{h}_{iT} = \begin{bmatrix} \frac{\mathbf{f}'\mathbf{s}_{i,-1}}{\sigma_i T^{3/2}} \\ \frac{\boldsymbol{\tau}'\mathbf{s}_{i,-1}}{\sigma_i T^{3/2}} \\ \frac{(\boldsymbol{\Upsilon}_1'\mathbf{s}_{i,-1})}{\sigma_i\sqrt{T}} \\ \frac{(\boldsymbol{\Upsilon}_2'\mathbf{s}_{i,-1})}{\sigma_i\sqrt{T}} \\ \frac{\mathbf{s}_{\Upsilon_1,-1}'\mathbf{s}_{i,-1}}{\sigma_i T^{3/2}} \\ \frac{\mathbf{s}_{\Upsilon_2,-1}'\mathbf{s}_{i,-1}}{\sigma_i T^{3/2}} \\ \frac{\mathbf{s}_{f,-1}'\mathbf{s}_{i,-1}}{\sigma_i T^{2}} \end{bmatrix}, \quad \mathbf{d}_{iT} = \begin{bmatrix} \mathbf{q}_{iT} \\ \frac{\mathbf{s}_{i,-1}'\boldsymbol{\varepsilon}_i}{\sigma_i T} \end{bmatrix}, \tag{14}
$$

$$
\boldsymbol{\Xi}_{iT}^{(1)} = \begin{bmatrix} \multicolumn{7}{c}{\boldsymbol{\Psi}_{fT}} \\ \frac{\mathbf{s}_{i,-1}'\mathbf{f}}{T^{3/2}} & \frac{\mathbf{s}_{i,-1}'\boldsymbol{\tau}}{T^{3/2}} & \frac{\mathbf{s}_{i,-1}'\boldsymbol{\Upsilon}_1}{\sigma_i T^{1/2}} & \frac{\mathbf{s}_{i,-1}'\boldsymbol{\Upsilon}_2}{\sigma_i T^{1/2}} & \frac{\mathbf{s}_{i,-1}'\mathbf{s}_{\Upsilon_1,-1}}{T^{3/2}} & \frac{\mathbf{s}_{i,-1}'\mathbf{s}_{\Upsilon_2,-1}}{T^{3/2}} & \frac{\mathbf{s}_{i,-1}'\mathbf{s}_{f,-1}}{T^{2}} \end{bmatrix},
$$

$$
\boldsymbol{\Xi}_{iT}^{(2)} = \begin{bmatrix} \frac{\mathbf{f}'\mathbf{s}_{i,-1}}{T^{3/2}} \\ \frac{\boldsymbol{\tau}'\mathbf{s}_{i,-1}}{T^{3/2}} \\ \frac{\boldsymbol{\Upsilon}_1'\mathbf{s}_{i,-1}}{\sigma_i T^{1/2}} \\ \frac{\boldsymbol{\Upsilon}_2'\mathbf{s}_{i,-1}}{\sigma_i T^{1/2}} \\ \frac{\mathbf{s}_{\Upsilon_1,-1}'\mathbf{s}_{i,-1}}{T^{3/2}} \\ \frac{\mathbf{s}_{\Upsilon_2,-1}'\mathbf{s}_{i,-1}}{T^{3/2}} \\ \frac{\mathbf{s}_{f,-1}'\mathbf{s}_{i,-1}}{T^{2}} \\ \frac{\mathbf{s}_{i,-1}'\mathbf{s}_{i,-1}}{T^{2}} \end{bmatrix}, \quad \boldsymbol{\Psi}_{fT}^{(1)} = \begin{bmatrix} \frac{\mathbf{f}'\mathbf{f}}{T} & \frac{\mathbf{f}'\boldsymbol{\tau}}{T} & \mathbf{f}'\boldsymbol{\Upsilon}_1 & \mathbf{f}'\boldsymbol{\Upsilon}_2 \\ \frac{\boldsymbol{\tau}'\mathbf{f}}{T} & 1 & \boldsymbol{\tau}'\boldsymbol{\Upsilon}_1 & \boldsymbol{\tau}'\boldsymbol{\Upsilon}_2 \\ \boldsymbol{\Upsilon}_1'\mathbf{f} & \boldsymbol{\Upsilon}_1'\boldsymbol{\tau} & T\boldsymbol{\Upsilon}_1'\boldsymbol{\Upsilon}_1 & T\boldsymbol{\Upsilon}_1'\boldsymbol{\Upsilon}_2 \\ \boldsymbol{\Upsilon}_2'\mathbf{f} & \boldsymbol{\Upsilon}_2'\boldsymbol{\tau} & T\boldsymbol{\Upsilon}_2'\boldsymbol{\Upsilon}_1 & T\boldsymbol{\Upsilon}_2'\boldsymbol{\Upsilon}_2 \\ \frac{\mathbf{s}_{\Upsilon_1,-1}'\mathbf{f}}{T} & \frac{\mathbf{s}_{\Upsilon_1,-1}'\boldsymbol{\tau}}{T} & \mathbf{s}_{\Upsilon_1,-1}'\boldsymbol{\Upsilon}_1 & \mathbf{s}_{\Upsilon_1,-1}'\boldsymbol{\Upsilon}_2 \\ \frac{\mathbf{s}_{\Upsilon_2,-1}'\mathbf{f}}{T} & \frac{\mathbf{s}_{\Upsilon_2,-1}'\boldsymbol{\tau}}{T} & \mathbf{s}_{\Upsilon_2,-1}'\boldsymbol{\Upsilon}_1 & \mathbf{s}_{\Upsilon_2,-1}'\boldsymbol{\Upsilon}_2 \\ \frac{\mathbf{s}_{f,-1}'\mathbf{f}}{T^{3/2}} & \frac{\mathbf{s}_{f,-1}'\boldsymbol{\tau}}{T^{3/2}} & \frac{\mathbf{s}_{f,-1}'\boldsymbol{\Upsilon}_1}{T^{1/2}} & \frac{\mathbf{s}_{f,-1}'\boldsymbol{\Upsilon}_2}{T^{1/2}} \end{bmatrix},
$$

$$
\boldsymbol{\Psi}_{fT}^{(2)} = \begin{bmatrix} \frac{\mathbf{f}'\mathbf{s}_{\Upsilon_1,-1}}{T} & \frac{\mathbf{f}'\mathbf{s}_{\Upsilon_2,-1}}{T} & \frac{\mathbf{f}'\mathbf{s}_{f,-1}}{T^{3/2}} \\ \frac{\boldsymbol{\tau}'\mathbf{s}_{\Upsilon_1,-1}}{T} & \frac{\boldsymbol{\tau}'\mathbf{s}_{\Upsilon_2,-1}}{T} & \frac{\boldsymbol{\tau}'\mathbf{s}_{f,-1}}{T^{3/2}} \\ \boldsymbol{\Upsilon}_1'\mathbf{s}_{\Upsilon_1,-1} & \boldsymbol{\Upsilon}_1'\mathbf{s}_{\Upsilon_2,-1} & \frac{\boldsymbol{\Upsilon}_1'\mathbf{s}_{f,-1}}{T^{1/2}} \\ \boldsymbol{\Upsilon}_2'\mathbf{s}_{\Upsilon_1,-1} & \boldsymbol{\Upsilon}_2'\mathbf{s}_{\Upsilon_2,-1} & \frac{\boldsymbol{\Upsilon}_2'\mathbf{s}_{f,-1}}{T^{1/2}} \\ \frac{\mathbf{s}_{\Upsilon_1,-1}'\mathbf{s}_{\Upsilon_1,-1}}{T} & \frac{\mathbf{s}_{\Upsilon_1,-1}'\mathbf{s}_{\Upsilon_2,-1}}{T} & \frac{\mathbf{s}_{\Upsilon_1,-1}'\mathbf{s}_{f,-1}}{T^{3/2}} \\ \frac{\mathbf{s}_{\Upsilon_2,-1}'\mathbf{s}_{\Upsilon_1,-1}}{T} & \frac{\mathbf{s}_{\Upsilon_2,-1}'\mathbf{s}_{\Upsilon_2,-1}}{T} & \frac{\mathbf{s}_{\Upsilon_2,-1}'\mathbf{s}_{f,-1}}{T^{3/2}} \\ \frac{\mathbf{s}_{f,-1}'\mathbf{s}_{\Upsilon_1,-1}}{T^{3/2}} & \frac{\mathbf{s}_{f,-1}'\mathbf{s}_{\Upsilon_2,-1}}{T^{3/2}} & \frac{\mathbf{s}_{f,-1}'\mathbf{s}_{f,-1}}{T^{2}} \end{bmatrix},
$$

in which $\boldsymbol{\varepsilon}_i = (\varepsilon_{i1}, \varepsilon_{i2}, ..., \varepsilon_{iT})'$, $\mathbf{f} = (f_1, f_2, ..., f_T)'$, $\mathbf{s}_{i,-1} = (0, s_{i1}, s_{i2}, ..., s_{i,T-1})'$, $\mathbf{s}_{f,-1} =$

$(s_{f0}, s_{f1}, ..., s_{f,T-1})'$, $\mathbf{s}_{\Upsilon_1,-1} = (0, s_{\Upsilon_1,1}, ..., s_{\Upsilon_1,T-1})'$, $\mathbf{s}_{\Upsilon_2,-1} = (0, s_{\Upsilon_2,1}, ..., s_{\Upsilon_2,T-1})'$ with $s_{\Upsilon_1,t} = \sum_{j=1}^{t} \Delta \sin(2\pi kj/T)$ and $s_{\Upsilon_2,t} = \sum_{j=1}^{t} \Delta \cos(2\pi kj/T)$.

Proof: See the Appendix.

The inclusion of Fourier terms in the null process results in the stochastic part of the regressors, $\Delta\bar{\mathbf{y}}$, $\bar{\mathbf{y}}_{-1}$ and $\mathbf{y}_{i,-1}$, being composed of the terms of different orders in probability under a fixed T. For example (see the Appendix for the detail), $\Delta\bar{\mathbf{y}}'\boldsymbol{\varepsilon}_i = (\alpha_1^* \boldsymbol{\Upsilon}_1' \boldsymbol{\varepsilon}_i)_{(\equiv O_p(T^{-1/2}))} + (\alpha_2^* \boldsymbol{\Upsilon}_2' \boldsymbol{\varepsilon}_i)_{(\equiv O_p(T^{-1/2}))} + (\gamma^* \mathbf{f}' \boldsymbol{\varepsilon}_i)_{(\equiv O_p(T^{1/2}))}$ and $\bar{\mathbf{y}}_{-1}'\boldsymbol{\varepsilon}_i = (\alpha_1^* \mathbf{s}_{\Upsilon_1,-1}' \boldsymbol{\varepsilon}_i)_{(\equiv O_p(T^{1/2}))} + (\alpha_2^* \mathbf{s}_{\Upsilon_2,-1}' \boldsymbol{\varepsilon}_i)_{(\equiv O_p(T^{1/2}))} + (\gamma^* \mathbf{s}_{f,-1}' \boldsymbol{\varepsilon}_i)_{(\equiv O_p(T))}$. Consequently, the scaling matrix $\boldsymbol{\Gamma}_T$ and $\boldsymbol{\Theta}_T$ in (11) and (13), respectively, are not diagonal matrices. Therefore neither can $\boldsymbol{\Gamma}_T$ be canceled out in $(\mathbf{q}_{iT}'\boldsymbol{\Gamma}_T')(\boldsymbol{\Gamma}_T \boldsymbol{\Psi}_{fT} \boldsymbol{\Gamma}_T')^{-1}(\boldsymbol{\Gamma}_T \mathbf{h}_{iT})$ and $(\mathbf{h}_{iT}'\boldsymbol{\Gamma}_T')(\boldsymbol{\Gamma}_T \boldsymbol{\Psi}_{fT} \boldsymbol{\Gamma}_T')^{-1}(\boldsymbol{\Gamma}_T \mathbf{h}_{iT})$ nor $\boldsymbol{\Theta}_T$ in $(\mathbf{d}_{iT}'\boldsymbol{\Theta}_T')(\boldsymbol{\Theta}_T \boldsymbol{\Xi}_{iT} \boldsymbol{\Theta}_T')^{-1}(\boldsymbol{\Theta}_T \mathbf{d}_{iT})$ from matrix inverse production. Hence, unlike the results of Pesaran (2007), the null distribution of $t_i(N,T)$ given by (7) is not free of nuisance parameters as $N \to \infty$ for any fixed T that is greater than 6 ($T > 6$). However, in the above example the stochastic trend terms ($\gamma^* \mathbf{f}' \boldsymbol{\varepsilon}_i$ and $\gamma^* \mathbf{s}_{f,-1}' \boldsymbol{\varepsilon}_i$) asymptotically dominate the other two deterministic trend terms ($\alpha_1^* \boldsymbol{\Upsilon}_1' \boldsymbol{\varepsilon}_i + \alpha_2^* \boldsymbol{\Upsilon}_2' \boldsymbol{\varepsilon}_i$ and $\alpha_1^* \mathbf{s}_{\Upsilon_1,-1}' \boldsymbol{\varepsilon}_i + \alpha_2^* \mathbf{s}_{\Upsilon_2,-1}' \boldsymbol{\varepsilon}_i$) as $T \to \infty$, resulting in the scaling matrix being a squared one and hence canceling out the effect of nuisance parameters in the distribution.[3] Simulation results in Section 4.1 and in the first panel of Table 3 below point out that the effect of nuisance parameters is negligible provided that $T \geq 30$.

Theorem 2:

Suppose y_{it} is generated based on (3) under the null hypothesis in (4) and Assumptions 1-4 hold. If next to N, T also tends to infinity (denoted as $(N,T)_{seq} \to \infty$), then the sequential limiting distribution of the statistic $t_i(N,T)$ defined by (7), referred to as the *BCADF* distribution, is :

$$BCADF_{if} = \frac{\int_0^1 W_i(r) dW_i(r) - \mathbf{q}_{if}' \boldsymbol{\Psi}_f^{-1} \mathbf{h}_{if}}{\left(\int_0^1 W_i^2(r) d(r) - \mathbf{h}_{if}' \boldsymbol{\Psi}_f^{-1} \mathbf{h}_{if}\right)^{1/2}}, \tag{15}$$

[3] The result is in contrast to that of West (1988) that includes a nonzero constant in the null unit-root process: $y_t = \alpha + y_{t-1} + \varepsilon_t$. In such a case, $\sum_{t=1}^{T} y_{t-1}\varepsilon_t = \sum_{t=1}^{T} [y_0 + \alpha(t-1) + s_{t-1}]\varepsilon_t$, where $s_t = \sum_{j=1}^{t} \varepsilon_j$. The deterministic trend term, $\sum_{t=1}^{T} \alpha(t-1)\varepsilon_t$, asymptotically dominates the stochastic trend term, $\sum_{t=1}^{T} s_{t-1}\varepsilon_t$. See Hamilton (1994, p. 496).

where

$$\mathbf{q}_{if} = \begin{bmatrix} W_i(1) \\ (2\pi k)\sigma[W(1) + 2\pi k \int_0^1 \sin(2\pi kr)W(r)d(r)] \\ (2\pi k)^2 \cdot \sigma[\int_0^1 \cos(2\pi kr)W(r)d(r)] \\ \int_0^1 W_f(r)dW_i(r) \end{bmatrix}, \tag{16}$$

$$\mathbf{h}_{if} = \begin{bmatrix} \int_0^1 W_i(r)dr \\ (2\pi k)\left(\int_0^1 W(s)ds + 2\pi k \int_0^1 \sin(2\pi kr)\left[\int_0^r W(s)ds\right]d(r)\right) \\ (2\pi k)^2\left(\int_0^1 \cos(2\pi kr)\left[\int_0^r W(s)ds\right]d(r)\right) \\ \int_0^1 W_f(r)W_i(r)dr \end{bmatrix}, \tag{17}$$

$$\mathbf{\Psi}_{f(4\times 4)} = \begin{bmatrix} \mathbf{A}_{2\times 2} & \mathbf{B} \\ \mathbf{B}' & \mathbf{D}_{2\times 2} \end{bmatrix}, \tag{18}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & (2\pi k)^2 \int_0^1 \cos^2(2\pi kr)dr \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{D}_1 & \mathbf{D}_2 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 0 & \int_0^1 W_f(r)dr \\ 0 & (2\pi k)\left(\int_0^1 W_f(s)ds + 2\pi k \int_0^1 \sin(2\pi kr)\left[\int_0^r W_f(s)ds\right]d(r)\right) \end{bmatrix},$$

$$\mathbf{D}_1 = \begin{bmatrix} (2\pi k)^2 \int_0^1 \sin^2(2\pi kr)dr \\ (2\pi k)^2\left(\int_0^1 \cos(2\pi kr)\left[\int_0^r W_f(s)ds\right]d(r)\right) \end{bmatrix},$$

$$\mathbf{D}_2 = \begin{bmatrix} (2\pi k)^2\left(\int_0^1 \cos(2\pi kr)\left[\int_0^r W_f(s)ds\right]d(r)\right) \\ \int_0^1 W_f^2(r)dr \end{bmatrix}.$$

$W_i(r)$ and $W_f(r)$ are independent standard Brownian motions.

Proof: See the Appendix.

The asymptotic distribution of $t_i(N,T)$ depends only on the frequency component, k, but is invariant to μ_i, $\alpha_{i,1}$, $\alpha_{i,2}$ and γ_i (as long as $\overline{\gamma} \neq 0$). Critical values of the individual *BCADF* statistic can be simulated based on 50,000 replications of regressing Δy_{it} on an intercept, $\Delta\sin(2\pi kt/T)$, $\Delta\cos(2\pi kt/T)$, $\Delta\overline{y}_t$, $\overline{y}_{t-1}$ and $y_{i,t-1}$ over the frequency $k = 1, ..., 5$ and the sample $t = 1, ..., T$. The individual series were generated based on $y_{it} = y_{i,t-1} + f_t + \varepsilon_{it}$ for $i = 1, ..., N$, and $t = -50, -49, ..., 1, 2, ..., T$ from $y_{i,-50} = 0$, with f_t and ε_{it} as *i.i.d.* $N(0,1)$.[4]

[4] These critical values are not reported in the paper but are available from the authors upon request.

It is interesting to examine the size distortion of Pesaran's (2007) $CIPS$ test in the presence of Fourier form breaks. Theoretically, Pesaran's test is subject to the problem of the omitted-variable bias when breaks exist. Although the bias vanishes when both N and T approach infinity as proved in the following corollary, the size distortion of Pesaran's t-statistic can be substantial in finite samples as indicated by the simulation results in Section 4.1.

Corollary:

Suppose Assumptions 1-4 hold and y_{it} is generated based on (3) under the null hypothesis in (4). Let $t_i^{P,B}(N,T)$ be the statistic for testing the unit-root hypothesis and it is the t-statistic of $\hat{b}_i$ in Pesaran's (2007) cross-sectionally augmented Dickey-Fuller regression: $\Delta y_{it} = c_{i,0} + c_{i,3}\overline{y}_{t-1} + c_{i,4}\Delta\overline{y}_t + b_i y_{i,t-1} + e_{it}$. Then, $t_i^{P,B}(N,T)$ has the following limiting distribution as $N \to \infty$:

$$t_i^{P,B}(N,T) \xrightarrow{N} \frac{\frac{\boldsymbol{\varepsilon}_i'\mathbf{s}_{i,-1}}{\sigma_i^2 T} - (\mathbf{q}_{iT}'\boldsymbol{\Gamma}_T'^P)(\boldsymbol{\Gamma}_T^P\boldsymbol{\Psi}_{fT}\boldsymbol{\Gamma}_T'^P)^{-1}(\boldsymbol{\Gamma}_T^P\mathbf{h}_{iT})}{J_1^P \times J_2^P + O_p(T^{-1/4})} + \frac{O_p(T^{-1/2})}{J_1^P \times J_2^P + O_p(T^{-1/4})},$$

where

$$J_1^P = \left(\frac{\boldsymbol{\varepsilon}_i'\boldsymbol{\varepsilon}_i}{\sigma_i^2(T-4)} - \frac{(\mathbf{d}_{iT}'\boldsymbol{\Theta}_T'^P)(\boldsymbol{\Theta}_T^P\boldsymbol{\Xi}_{iT}\boldsymbol{\Theta}_T'^P)^{-1}(\boldsymbol{\Theta}_T^P\mathbf{d}_{iT})}{T-4}\right)^{1/2}, \tag{19}$$

$$J_2^P = \left(\frac{\mathbf{s}_{i,-1}'\mathbf{s}_{i,-1}}{\sigma_i^2 T^2} - (\mathbf{h}_{i,T}'\boldsymbol{\Gamma}_T'^P)(\boldsymbol{\Gamma}_T^P\boldsymbol{\Psi}_{fT}\boldsymbol{\Gamma}_T'^P)^{-1}(\boldsymbol{\Gamma}_T^P\mathbf{h}_{iT})\right)^{1/2}, \tag{20}$$

$$\boldsymbol{\Gamma}_T^P = \begin{bmatrix} \gamma^* & 0 & T^{-1}\alpha_1^* & T^{-1}\alpha_2^* & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & T^{-1/2}\alpha_1^* & T^{-1/2}\alpha_2^* & \gamma^* \end{bmatrix}, \tag{21}$$

$$\boldsymbol{\Theta}_T^P = \begin{bmatrix} & & & \boldsymbol{\Gamma}_T^P & & & & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0 & T^{-1/2}y_0 & 0 & 0 & T^{-1/2}\alpha_{i,1} & T^{-1/2}\alpha_{i,2} & \gamma_i & 1 \end{bmatrix}, \tag{22}$$

and $\mathbf{q}_{iT}$, $\mathbf{h}_{iT}$, $\mathbf{d}_{iT}$ and $\boldsymbol{\Psi}_{fT}$ and $\boldsymbol{\Xi}_{iT}$ are defined in (12) and (14), respectively. If next to N, T also tends to infinity, then

$$t_i^{P,B}(N,T) \xrightarrow{(N,T)_{seq}} \frac{\int_0^1 W_i(r)dW_i(r) - \mathbf{q}_{if}^{P\prime}\left(\boldsymbol{\Psi}_f^P\right)^{-1}\mathbf{h}_{if}^P}{\left(\int_0^1 W_i^2(r)d(r) - \mathbf{h}_{if}^{P\prime}\left(\boldsymbol{\Psi}_f^P\right)^{-1}\mathbf{h}_{if}^P\right)^{1/2}},$$

where

$$\mathbf{q}_{if}^{P} = \begin{bmatrix} W_i(1) \\ \int_0^1 W_f(r)dW_i(r) \end{bmatrix}, \ \mathbf{h}_{if}^{P} = \begin{bmatrix} \int_0^1 W_i(r)dr \\ \int_0^1 W_f(r)W_i(r)dr \end{bmatrix},$$

$$\mathbf{\Psi}_{f}^{P} = \begin{bmatrix} 1 & \int_0^1 W_f(r)dr \\ \int_0^1 W_f(r)dr & \int_0^1 W_f^2(r)dr \end{bmatrix}.$$

The expression: $\frac{\int_0^1 W_i(r)dW_i(r)-\mathbf{q}_{if}^{P\prime}\left(\mathbf{\Psi}_f^P\right)^{-1}\mathbf{h}_{if}^P}{\left(\int_0^1 W_i^2(r)d(r)-\mathbf{h}_{if}^{P\prime}\left(\mathbf{\Psi}_f^P\right)^{-1}\mathbf{h}_{if}^P\right)^{1/2}}$ is the limiting distribution of Pesaran's (2007) $CADF$ statistic, constructed from the cross-sectionally augmented regression, for testing the unit-root hypothesis as $N, T \to \infty$ when there is no break in DGP.

$\mathfrak{Proof}$: See the Appendix.

The limiting distribution of $t_i^{P,B}(N,T)$ under a fixed T includes the $O_p(T^{-1/2})$ and $O_p(T^{-1/4})$ terms, which are the finite-sample bias of slope and standard error estimation, respectively, resulting from omitted breaks in Pesaran's cross-sectionally augmented regression. It is straightforward to show that $O_p(T^{-1/4})$ is positive but $O_p(T^{-1/2})$ can be either positive or negative depending on the relative influence of the factor loading (γ_i) and Fourier term parameters ($\alpha_{i,1}$ and $\alpha_{i,2}$). Therefore, Pesaran's $CADF$ test can either under- or over-reject the unit-root hypothesis in finite samples when smooth breaks exist.

3.2 BCADF−based Panel Unit-Root Tests

The null distribution of the individual $BCADF$ statistic is asymptotically independent of nuisance parameters when both N and T go to infinity. However, $BCADF_{if}$ and $BCADF_{jf}$ are dependently distributed with the same degree of dependence for all $i \neq j$ since they are all nonlinear function of a common process W_f. To develop a panel unit-root test, this paper considers the breaks and cross-sectional dependence augmented version of the IPS test as follows:

$$BCIPS(N,T) = \frac{1}{N}\sum_{i=1}^{N} t_i(N,T).$$

To ensure $t_i(N,T)$ having finite moments, we follow Pesaran (2007) to construct the truncated version of the $BCIPS$ statistic:

$$BCIPS^*(N,T) = \frac{1}{N}\sum_{i=1}^{N} t_i^*(N,T), \tag{23}$$

where

$$\begin{aligned} t_i^*(N,T) &= t_i(N,T), \quad if \quad -M_1 < t_i(N,T) < M_2, \\ t_i^*(N,T) &= -M_1, \qquad if \;\; t_i(N,T) \le -M_1, \\ t_i^*(N,T) &= M_2, \qquad if \;\; t_i(N,T) \ge M_2, \end{aligned}$$

M_1 and M_2 are two positive constants such that $Pr(-M_1 < t_i(N,T) < M_2)$ is sufficiently large. Following Pesaran's (2007) arguments for the convergence of $CIPS^*(N,T)$, it can be shown that $BCIPS^*(N,T)$ converges almost surely to a distribution depending on M_1, M_2 and W_f.[5] Although the limiting distribution of $BCIPS^*(N,T)$ is not analytically tractable, it can be readily simulated using (23). Using the normal approximation of $t_i(N,T)$, we compute

$$M_1 = -E(BCADF_{if}) - \Phi^{-1}(\epsilon/2)\sqrt{Var(BCADF_{if})}, \tag{24}$$

$$M_2 = -E(BCADF_{if}) + \Phi^{-1}(1-\epsilon/2)\sqrt{Var(BCADF_{if})}, \tag{25}$$

where Φ^{-1} is the inverse of the cdf of the normal distribution, and $BCADF_{if}$ is the stochastic limit of $t_i(N,T)$ as N and T tend to infinity such that $N/T \to d$, $(0 < d < \infty)$. After constructing $E(BCADF_{if})$ and $Var(BCADF_{if})$ by simulation and setting $\epsilon = 10^{-6}$, we calculate M_1 and M_2 across different k under three different models and report them in Table 1. The three models under consideration are the model without intercept and trend, with intercept, and with intercept and trend, respectively. This paper simulates M_1 and M_2 across five different k by setting $N = 300$ and $T = 500$ with $50,000$ replications.

< Table 1 here >

Tables 2(a) to 2(c) report 1%, 5% and 10% critical values of $BCIPS$ and $BCIPS^*$ statistics for three different models across different k, N and T.[6] The critical values of the

[5] Since the included Fourier terms are deterministic functions which can only affect the conditional expectation of Pesaran's (2007) $CADF^*_{if}$, i.e. $E(CADF_{1f}|W_f)$, it is appropriate to discuss the convergence of $BCIPS^*(N,T)$ by following Pesaran's (2007) arguments for the convergence of $CIPS^*(N,T)$.

[6] We also simulate critical values based on the following DGP: $y_{it} = y_{i,t-1} + \gamma_i f_t + \alpha_{i,1}\Delta\sin(2\pi kt/T) + \alpha_{i,2}\Delta\cos(2\pi kt/T) + \varepsilon_{it}$, where $\gamma_i \sim i.i.d.U[-1,3]$ and $\alpha_{i,1}, \alpha_{i,2} \sim i.i.d.U[3,5]$ and $\sim i.i.d.U[-2,2]$, respectively.

$BCIPS$ statistic may differ from those of $BCIPS^*$ when T is less than 50 and both critical values are reported with those of $BCIPS^*$ in parentheses, otherwise only the critical values of the $BCIPS$ statistic are reported. It is worth noting that if critical values of $BCIPS^*$ and $BCIPS$ are different, then the value of the former statistic is slightly larger than that of the latter. This indicates a slightly rightward shift of the null distribution of the $BCIPS^*$ statistic relative to that of $BCIPS$. Besides, critical values reported in Tables 2(a) - 2(c) are also lower than the corresponding values of the $CIPS$ and $CIPS^*$ statistics in Pesaran (2007) that neglects smooth changes in deterministic terms.

< Tables 2(a), 2(b), (2c) here >

3.3 The Case with Serially Correlated Errors

Our discussion in Sections 3.1 and 3.2 can be extended to the case where individual-specific errors are serially correlated. Following Pesaran (2007), two different specifications for serially correlated errors are given as follows:

$$
\begin{aligned}
u_{it} &= \rho_i u_{i,t-1} + \eta_{it}, \qquad \eta_{it} = \gamma_i f_t + \varepsilon_{it}; & (26)\\
u_{it} &= \gamma_i f_t + \eta_{it}, \qquad \eta_{it} = \rho_i \eta_{i,t-1} + \varepsilon_{it}, & (27)
\end{aligned}
$$

where ε_{it} is the idiosyncratic error. To derive the asymptotic distribution of the $BCADF$ statistic, we focus our discussion on the specification of (26) and assume homogeneous autoregressive coefficients across i ($\rho_i = \rho$) to simplify the mathematical derivation. However, heterogeneous autoregressive coefficients are allowed in Monte-Carlo experiments. The asymptotic distribution to be derived in this section can be adapted to deal with both specifications in (26) and (27).

Replacing u_{it} in (1) with (26) and imposing the above-mentioned homogeneity assumption, we obtain:

$$
\begin{aligned}
\Delta y_{it} &= \boldsymbol{\alpha}_i'\Delta\mathbf{d}_t + \beta_i(1-\rho)(y_{i,t-1} - \boldsymbol{\alpha}_i'\mathbf{d}_{t-1}) + \rho(1+\beta_i)(\Delta y_{i,t-1} - \boldsymbol{\alpha}_i'\Delta\mathbf{d}_{t-1})\\
&\quad + \gamma_i f_t + \varepsilon_{it}. \qquad (28)
\end{aligned}
$$

To test the null hypothesis in (4), this paper estimates the following breaks and cross-sectional

The critical values in Tables 2(a)-2(c) are barely affected by these changes. These results are available from the authors upon request.

dependence augmented ADF regression:

$$\Delta y_{it} = c_{i,0} + c_{i,1}\Delta\sin(2\pi kt/T) + c_{i,2}\Delta\cos(2\pi kt/T) + c_{i,3}\overline{y}_{t-1} + c_{i,4}\Delta\overline{y}_t + c_{i,5}\Delta y_{i,t-1}$$
$$+c_{i,6}\Delta\overline{y}_{t-1} + b_i y_{i,t-1} + e_{it}.$$

The t-statistic of the estimate of b_i $(\hat{b}_i)$ is then applied to examine the unit-root hypothesis and it can be written as

$$t_i^{\rho}(N,T) = \frac{\Delta\mathbf{y}_i'\mathbf{M}_z^{\rho}\mathbf{y}_{i,-1}}{\hat{\sigma}_i(\mathbf{y}_{i,-1}'\mathbf{M}_z^{\rho}\mathbf{y}_{i,-1})^{1/2}}, \tag{29}$$

where $\hat{\sigma}_i^2 = \frac{\Delta\mathbf{y}_i'\mathbf{M}_{i,z}^{\rho}\Delta\mathbf{y}_i}{T-8}$, $\mathbf{M}_z^{\rho} = \mathbf{I}_T - \mathbf{Z}^{\rho}(\mathbf{Z}^{\rho\prime}\mathbf{Z}^{\rho})^{-1}\mathbf{Z}^{\rho\prime}$, $\mathbf{Z}^{\rho} = (\Delta\mathbf{y}_{i,-1}, \Delta\overline{\mathbf{y}}_{-1}, \Delta\overline{\mathbf{y}}, \boldsymbol{\tau}, \boldsymbol{\Upsilon}_1, \boldsymbol{\Upsilon}_2, \overline{\mathbf{y}}_{-1})$, $\mathbf{M}_{i,z}^{\rho} = \mathbf{I}_T - \mathbf{G}_i^{\rho}(\mathbf{G}_i'^{\rho}\mathbf{G}_i^{\rho})^{-1}\mathbf{G}_i'^{\rho}$, $\mathbf{G}_i^{\rho} = (\mathbf{Z}^{\rho}, \mathbf{y}_{i,-1})$. The exact sampling distribution of $t_i^{\rho}(N,T)$ is affected by $\alpha_{i,1}, \alpha_{i,2}, \gamma_i$ and ρ, but as stated in the following theorem their affection disappears as $(N,T)_{seq} \to \infty$.

Theorem 3:

Suppose y_{it} is generated by (28) with $|\rho| < 1$ under the null hypothesis of $\beta_i = 0$ for all i and assumptions 1-4 hold. Let $t_i^{\rho}(N,T)$ be the t-statistic defined in (29). Then, as $(N,T)_{seq} \to \infty$, $t_i^{\rho}(N,T)$ has the same sequential limiting distribution given by (15), obtained under $\rho = 0$.

Proof: See the Appendix.

Since $t_i^{\rho}(N,T)$ in (29) has the same limiting distribution as that of (15), the panel unit-root test, $BCIPS^*(N,T)$, proposed in the previous section can be applied equally to the case with serially correlated errors. For an $AR(p)$ specification of errors in (26),

$$(1 - \rho_{i,1}L - \cdots - \rho_{i,p}L^p)u_{it} = \eta_{it}, \quad \eta_{it} = \gamma_i f_t + \varepsilon_{it},$$

in which all roots of $(1 - \rho_{i,1}z - \cdots - \rho_{i,p}z^p) = 0$ lie outside the unit circle and $\varepsilon_{it} \sim N(0, \sigma_i^2)$. We suggest the following $BCADF$ regressions for the level and trend cases, respectively,

$$
\begin{aligned}
\Delta y_{it} &= c_{i,0} + c_{i,1}\Delta\sin(2\pi kt/T) + c_{i,2}\Delta\cos(2\pi kt/T) + c_{i,3}\overline{y}_{t-1} + \sum_{j=0}^{p} d_{ij}\Delta\overline{y}_{t-j} \\
&\quad + \sum_{j=1}^{p}\delta_{ij}\Delta y_{i,t-j} + b_i y_{i,t-1} + e_{it}, \qquad (30) \\
\Delta y_{it} &= c_{i,0} + \tau\cdot t + c_{i,1}\Delta\sin(2\pi kt/T) + c_{i,2}\Delta\cos(2\pi kt/T) + c_{i,3}\overline{y}_{t-1} + \sum_{j=0}^{p} d_{ij}\Delta\overline{y}_{t-j} \\
&\quad + \sum_{j=1}^{p}\delta_{ij}\Delta y_{i,t-j} + b_i y_{i,t-1} + e_{it}. \qquad (31)
\end{aligned}
$$

4 Finite Sample Performance

The data generating process is given as follows:

$$
y_{it} = \mu_{i,1}(1-\phi_i L)t + (1-\phi_i L)\varpi_{i,k,t} + \phi_i y_{i,t-1} + u_{it},\ i = 1,2,...,N,\ t = 1,2,..,T, \quad (32)
$$

where $\varpi_{i,k,t} = \mu_i + \alpha_{i,1}\sin(2\pi kt/T) + \alpha_{i,2}\cos(2\pi kt/T);\ u_{it} = \gamma_i f_t + \eta_{it},\ \eta_{it} = \rho_i\eta_{i,t-1} + \varepsilon_{it},\ y_0, \mu_{i,1}, \mu_i \sim i.i.d.N(0,1)$. Following Pesaran (2007), we set $f_t \sim i.i.d.N(0,1)$, $\varepsilon_{it} \sim i.i.d.N(0,\sigma_i^2)$ with $\sigma_i^2 \sim i.i.d.U[0.5,1.5]$, $\rho_i \sim i.i.d.U[0.2,0.4]$ and $\rho_i \sim i.i.d.U[-0.4,-0.2]$ to denote the case of positive and negative residual serial correlations, respectively, $\gamma_i \sim i.i.d.U[-1,3]$ and $\gamma_i \sim i.i.d.U[0,0.2]$ to denote the case of high and low cross-sectional dependence, respectively, and $\alpha_{i,1}, \alpha_{i,2} \sim i.i.d.U[0,0.2]$ and $\alpha_{i,1}, \alpha_{i,2} \sim i.i.d.U[1,2]$ to denote the case of the low and high amplitudes of breaks, respectively.

The specification of residual in (32) differs from (26) that is used to derive Theorem 3 and the purpose of this change is to check the robustness of our analysis to an alternative residual serial-correlation specification. Equation (32) degenerates to the intercept case if $\mu_{i,1} = 0$ for all i and it degenerates to the case with serially uncorrelated errors if $\rho_i = 0$ for all i. Sizes are computed under the null hypothesis of $\phi_i = 1$ for all i. Powers are constructed under the alternative hypothesis of $\phi_i \sim i.i.d.U[0.85, 0.95]$. The common factor ($f_t$) was generated independently of ε_{it} and the parameters $\phi_i, \mu_i, \mu_{i,1}, \alpha_{i,1}, \alpha_{i,2}, \gamma_i, \rho_i, \sigma_i$ were also drawn independently of ε_{it}. The tests were one-sided with the nominal size set at 5%, and were conducted for $N = 10, 20, 30, 50, 100, 200$, $T = 30, 50, 100, 200, 400$ and $k = 1, 2, 3$. The size and power for each experiment were constructed using $5,000$ replications. Critical values for different combinations of k, N and T under three different models, reported in Tables

2(a)-2(c), are adopted to examine the size and power of the $BCIPS^*$ statistic. The size and power of the $BCIPS$ statistic are not reported but are available from the authors upon request.

4.1 Size and Power under Serially Uncorrelated Errors

Before discussing the size and power of our proposed test, it would be helpful to discuss the size distortion of Pesaran's (2007) $CIPS$ test when smooth breaks exist. To emphasize the role of the amplitude of breaks, size distortions are examined for the large and small amplitude of breaks under different frequency components (k). The sizes of $CIPS^*$ test are generally close to 0.05 regardless of N, T and k when the magnitude of breaks is small. However, the $CIPS^*$ test can be very conservative when the magnitude of breaks is large since the sizes, in such a case, are generally very small even when $T = 400$, but they are close to 0.05 when $T = 5,000$.[7] The above results are not reported but are available from the authors upon request. Recall that Pesaran's $CADF$ test is biased when breaks exist, but the distortion disappears when N and $T \to \infty$ as pointed out by the Corollary in Section 3.1. Our simulation results support the above theoretical finding. Given that the size distortion of the $CIPS^*$ test is serious with large amplitudes of smooth breaks, this paper therefore focuses its discussion under the case of $\alpha_{i,1}, \alpha_{i,2} \sim i.i.d.U[1, 2]$.

As a benchmark, we begin by examining the finite-sample size and power of the $BCIPS^*$ test for various values of k and assume it is known in estimation. The size of the test with an unknown k is discussed in Section 4.4. The size and power of the $BCIPS^*$ test for high cross-sectional dependence and large amplitudes of breaks are reported in Table 3. The results from the first panel of Table 3 indicate that the size of the $BCIPS^*$ statistic over different N, T and k are in general close to 0.05 (with a minimum of 0.043 and a maximum of 0.071) as long as $T \geq 30$. As for the power, the results from the second panel of Table 3 indicate that the power is poor when $T \leq 30$. With a sample size of fifty, the power of the $BCIPS^*$ test is generally high when $N > 20$ and $k > 1$. Under a given N, the power of the $BCIPS^*$ test increases with T significantly and it is close to 1.0 for most cases when $T > 50$. It is interesting to find that, when T is very small (T=15), the size of the $BCIPS^*$ statistic is a little bit high in general even when N is large. Recall that Theorem 1 indicates that

[7]This paper also considers two different cases for the large amplitude of breaks: $\alpha_{i,1}, \alpha_{i,2} \sim i.i.d.U[-2, 2]$ and $\alpha_{i,1}, \alpha_{i,2} \sim i.i.d.U[3, 5]$. The sizes of the $CIPS^*$ test in the former case increase with T and are close to 0.05 when $T = 5,000$ but the sizes in the latter case vary with T with a non-monotonic pattern and are generally close to 0.05 when $T = 10,000$. The above results are available from the authors upon request.

the $BCADF$ distribution is not free of nuisance parameters in finite T even when $N \to \infty$. The finding of relatively high sizes, with large N, when T is very small ($T = 15$) agrees with Theorem 1. Moreover, the results from the first panel of Table 3 indicate that the influence of nuisance parameters on the $BCADF$ distribution is negligible as long as $T \geq 30$. In short, the results from Table 3 indicate that, by allowing for high cross dependence and large amplitudes of breaks, the size of the $BCIPS^*$ test is generally reasonable when $T \geq 30$ and the power of the test is high when $T > 50$.[8]

< Table 3 here >

4.2 Size and Power when Residuals are Serially Correlated

In the case of existing first-order autoregressive errors, we consider the scenarios of positive and negative serial correlations. With a positive residual serial correlation, the sizes of the $BCIPS^*$ test are close to 0.05 when $k = 1$ and they are generally reasonable when $k = 2, 3$ and $T > 50$ as indicated by the first panel of Table 4. With a negative residual serial correlation, the results from the second panel of Table 4 point out that the size is generally close to 0.05 (with a minimum of 0.041 and a maximum of 0.070) regardless of N and T when $k \leq 2$. As for the case of $k = 3$, the size of the $BCIPS^*$ test is reasonable except for those cases with $T = 30$. In short, the sizes of the $BCIPS^*$ test, under the case of a positive (negative) residual serial correlation, are generally reasonable when $T > 50$ ($T > 30$). The results from the third panel of Table 4 indicate that the power of the $BCIPS^*$ test under a positive residual serial correlation is generally high when $T > 50$ regardless of N and k. Similar results are observed when the residual serial correlation is negative as pointed out in the fourth panel of Table 4.

< Tables 4 here >

4.3 Size and Power under the Linear Trend Model

It is beneficial to first examine the performance of our proposed test when DGP includes a linear trend and large amplitudes of smooth breaks in the mean under the case of serially uncorrelated errors. The results from Table 5 indicate that the sizes of the $BCIPS^*$ statistic are generally reasonable and close to 0.05. Besides, the power of the statistic is generally higher than 0.7 when the $T > 50$. The above results are generally similar to those found in Table 3.

[8]Similar results are obtained when the amplitude of breaks is either $\alpha_{i,1}, \alpha_{i,2} \sim i.i.d.U[-2, 2]$ or $\alpha_{i,1}, \alpha_{i,2} \sim i.i.d.U[3, 5]$, which are available from the authors upon request.

With a positive residual serial correlation, the results from the first panel of Table 6 indicate that the $BCIPS^*$ test has reasonable sizes when $k < 3$. In the case of $k = 3$, the sizes of the $BCIPS^*$ test are generally reasonable when $T > 50$. As for the case of a negative residual serial correlation, the results from the second panel of Table 6 reveal that the size of our proposed test is close to 0.05 regardless of N, T and k. As for the power of the $BCIPS^*$ test, they are reasonably high in general when $T > 50$ regardless of the residual serial correlation being positive or negative as indicated by the third and fourth panels of Table 6.

< Tables 5, 6 here >

4.4 Test with k Unknown

In the case of serially uncorrelated errors with an unknown k, we first estimate the unknown Fourier frequency component (k) from the data and then employ the estimated k ($\hat{k}$) to evaluate the size and power of the $BCIPS^*$ test. The determination of k is based on the grid-search method suggested by Davis (1987). Following Enders and Lee (2009), this paper sets the maximum Fourier frequency component to k^{max}=5 and then estimate (31) with the lag order being zero ($p = 0$) for a given k. After estimating the above equation, we compute the sum of squared residuals ($SSR_{k,i}$), $i = 1, ..., N$, $k = 1, ..., 5$ under a given N and T. The estimated k is obtained by minimizing the sum of $SSR_{k,i}$ across i: $\hat{k} = \min_{\{k\}} \sum_{i=1}^{N} SSR_{k,i}$. The $BCIPS^*$ statistic is then calculated as before based on $\hat{k}$ and its sizes under different N and T are reported in Table 7. For the intercept case, the results from the first panel of Table 7 point out that the sizes of the $BCIPS^*$ test are generally reasonable and close to 0.05 regardless of N, T and k. As for the linear trend case, the sizes of the $BCIPS^*$ test are close to 0.05 when $T > 30$ except for the cases of k=1, 2 and $N \leq 20$.

< Tables 7, 8 here >

5 Empirical Application

An important characteristic of real exchange rates under the recent float is their long swings (Engel and Hamilton, 1990) resulting from common shocks or long-lived bubbles. There are several significant common shocks under the period of recent float, which create long-lived bubbles affecting the nominal exchange rates of many countries.[9] Papell (2002) argues that

[9]These common shocks include the stock market crash of 1973-1974, the UK's secondary banking crisis of 1973-1975, the Latin American debt crisis of 1982, the Plaza Accord of 1985, the US's saving and loan crisis

the failure to reject long-run PPP in most of the existing literature could be due to the ignorance of smooth breaks in real exchange rates. Although the conventional literature modeled real exchange rates with a non-trend model, several articles have pointed out the significance of the Balassa-Samuelson effects over the post-Bretton Woods period (Bergin et al., 2006; Wu and Hu, 2009; Paya et al., 2003; Sollis, 2005). This paper, therefore, applies a trended $BCADF$ regression as indicated by (31) to investigate the validity of PPP.

Both monthly and quarterly nominal exchange rates and consumer price indices (CPI) for sixty countries over the post-Bretton Woods period are downloaded from the IMF's International Financial Statistics. The sample period starts in January 1974 and ends in April 2011. For euro-zone countries, the dollar-based nominal exchange rates after 1999 were constructed by using the euro-dollar rate and the prefixed exchange rates at January 1st, 1999 (Alba and Papell, 2007). Australia and New Zealand do not report monthly CPI figures and hence they are converted from quarterly CPI with quadratic-match averages (Molodtsova and Papell, 2009). Two different panels are considered in the empirical investigation: P60 including sixty countries from the world and P39 including thirty-nine countries in the panel of P60.

To determine the frequency component (k) in the Fourier function, we first allow the frequency component to vary from one to five and then estimate equation (31) for each value of k under a given lag order (p). This paper then constructs the sum of square errors ($SSR_{k,p,i}$) for the ith country under a given k and p. The estimate of k ($\hat{k}$) under a given lag order, p, is obtained by choosing a specific k that minimizes the sum of $SSR_{k,p,i}$ over i. The maximum lag order (p^{max}) in (31) is set to 12 for monthly data and 4 for quarterly data.

After estimating the frequency component ($\hat{k}$) in the Fourier function, the $BCIPS^*$ statistic is constructed by estimating equation (31) under different lag orders. With monthly data, the results from the first panel of Table 8 indicate that the estimated frequency component ($\hat{k}$) in the Fourier function is one regardless of the lag order p. This paper then estimates an ADF equation augmented with the Fourier function in which the frequency component is set to one under a given p, and then examines the significance of the cross dependence across estimated residuals by the cross-sectional dependence (CD) test of Pesaran (2004). The results from the first panel of Table 8 point out that the CD test is significant at conventional levels and the mean of the pair-wise cross-correlation coefficient of estimated residuals is about 0.17 for the panel of P60. The $BCIPS^*$ statistic, based on the panel of P60, rejects the unit-root

of 1989-1991, the European exchange rate mechanism crisis of 1992-1993, the Mexican crisis of 1994-1995, the East Asian crisis of 1997-98, the technology bubbles of 2000-01, the Argentinean crisis of 2001-02, the global financial crisis of 2007-10 and the European debt crisis of 2010-2011.

hypothesis at conventional levels for all lags. Besides, the average of the estimated amplitude of the sine (Ave(c1)) and cosine terms (Ave(c2)) in the Fourier function, under different lag orders, changes in the interval of (-0.030, -0.002) and (-0.013, 0.048), respectively. However, the $CIPS^*$ statistic rejects the unit-root hypothesis for only five out of twelve lags. As for the panel of P39, the results from the second panel of Table 8 indicate significant cross-sectional dependence. The $BCIPS^*$ statistic rejects the unit-root hypothesis for ten out of twelve lags but the $CIPS^*$ statistic fails to rejects the hypothesis for all lags. The average of the estimated amplitude of the sine (Ave(c1)) and cosine terms (Ave(c2)) in the Fourier function changes in the interval of (-0.050, -0.028) and (-0.041, 0.014), respectively.

As for the quarterly data, the results from the third panel of Table 8 again indicate that the estimated frequency component in the Fourier function ($\hat{k}$) is one and that Pesaran's (2004) CD test reveals significant cross-sectional dependence regardless of the panel being P60 or P39. The $BCIPS^*$ statistic rejects the unit-root hypothesis at conventional levels for all lags regardless of the panel being P60 or P39. The $CIPS^*$ statistic rejects the hypothesis for two out of four lags for the panel of P60, but fails to reject the hypothesis for all lags for the panel of P39.[10] It is interesting to find that the $BCIPS^*$ statistic reveals stronger evidence of rejecting the unit-root hypothesis than the $CPIS^*$ statistic. A reason for the above finding should be that the amplitudes of smooth breaks measured by the sine and cosine terms in the Fourier function are significant. This paper, therefore, examines if the estimated coefficients of the sine and cosine terms are jointly insignificant. The null hypothesis is $\alpha_{i,1} = \alpha_{i,2} = 0$ for all $i \in N$. This paper constructs the average of the F statistic across individuals and refers it to the AF statistic. We then construct the finite sample distribution of AF by a block bootstrap (Kunsch, 1989).[11] The block size is set in accordance with the lag order in the $BCADF$ regression equation.

Testing the hypothesis of $\alpha_{i,1} = \alpha_{i,2} = 0$ with a F test is appropriate only when the unit-root hypothesis is rejected. We therefore examine the above hypothesis for those lag orders that the $BCIPS^*$ statistic rejects the unit-root hypothesis. The null hypothesis of $\alpha_{i,1} = \alpha_{i,2} = 0$ for all i is rejected at conventional levels for both panels regardless of the

[10] We also select a model across k and p that has the minimum SSR and results reveal that the $BCIPS^*$ test rejects the unit-root hypothesis but the $CIPS^*$ test fails to reject the hypothesis regardless of the panel being P60 or P39 and of the data frequency being monthly or quarterly.

[11] After generating a bootstrap sample based on the block bootstrap, we estimate the individual BCADF equation and then construct the $F_i^{(b)}$ statistic for the hypothesis of $\alpha_{i,1} = \alpha_{i,2} = 0$. The $AF^{(b)}$ statistic is then obtained by averaging $F_i^{(b)}$, i.e., $AF^{(b)} = \frac{1}{N}\sum_{i=1}^{N} F_i^{(b)}$. By repeating the above procedure for $b = 1,000$ times, this paper constructs the finite sample distribution of the AF statistic.

data frequency being monthly or quarterly. The results from Table 8 are interesting since they point out that the estimated amplitudes of smooth breaks are significant and neglecting these breaks results in the failure to reject the unit-root hypothesis. In sum, by taking into account smooth breaks, our proposed test strongly rejects the unit-root hypothesis but Pesaran's panel unit-root test reveals only weak evidence to reject the hypothesis. These results indicate that controlling smooth breaks in the mean is crucial to examine the validity of long-run PPP over the post-Bretton Woods period.

6 Conclusion

This paper develops a panel unit-root test that accommodates cross-sectional dependence among variables and smooth structural changes in deterministic components. Applying the sequential limit approach, this paper first shows that the asymptotic distribution of the individual $BCADF$ statistic is free of nuisance parameters when both T and N approach infinity. The proposed panel unit-root test, $BCIPS^*$, is therefore the breaks and cross dependence augmented version of the IPS test. The limiting distribution of the $BCIPS$ statistic is shown to exist and its critical values are tabulated. Finite-sample properties of the $BCIPS$ are then investigated by Monte-Carlo simulations. With serially uncorrelated errors, simulation results indicate that the size of the $BCIPS^*$ test is generally reasonable regardless of N, and k when $T \geq 30$. With serially correlated errors, the size and power of the $BCIPS^*$ statistic are generally reasonable regardless of N, k and the residual serial correlation being positive or negative as long as $T > 50$. Finally, our proposed test and Pesaran's (2007) panel unit-root test are applied to test the validity of long-run PPP. The empirical results point out that controlling smooth breaks in the mean is crucial to the validity of long-run PPP.

Appendix

The following two Lemmas collect the large sample behavior of the scaled product involving Fourier terms that are used to prove Theorems 2 and 3 and the Corollary below.

Lemma 1:

(L1): $T\sum_{t=1}^{T}\Delta\sin^2(2\pi kt/T)\xrightarrow{T}(2\pi k)^2\int_0^1\cos^2(2\pi kr)dr$,

(L2): $T\sum_{t=1}^{T}\Delta\cos^2(2\pi kt/T)\xrightarrow{T}(2\pi k)^2\int_0^1\sin^2(2\pi kr)dr$,

(L3): $\sum_{t=1}^{T}\Delta\sin(2\pi kt/T)\Delta\cos(2\pi kt/T)=0$,

(L4): $\sum_{t=1}^{T}\Delta\sin(2\pi kt/T)=\sum_{t=1}^{T}\Delta\cos(2\pi kt/T)=0$,

(L5): $\Delta\sin(2\pi kt/T)=2\pi k/T\cos(2\pi kt/T)$,

(L6): $\Delta\cos(2\pi kt/T)=-2\pi k/T\sin(2\pi kt/T)$,

(L7): $1/T\sum_{t=1}^{T}\sin(2\pi kt/T)\cos(2\pi k(t-1)/T)\to 0$.

Lemma 2: Let z_t be a serially correlated as well as heterogeneously distributed innovation satisfying the following Functional Central Limit Theorem:[12] $W_T(r)=T^{-1/2}\sum_{t=1}^{[Tr]}z_t/\sigma\Rightarrow W(r)$, for $r\in[0,1]$, where $W(r)$ is a standard Brownian Motion and $\sigma^2=\lim_{T\to\infty}T^{-1}$ $E\left(\sum_{t=1}^{T}z_t\right)^2$. Then

(L8): $\sqrt{T}\sum_{t=1}^{T}z_t\Delta\sin(2\pi kt/T)\xrightarrow{T}(2\pi k)\sigma[W(1)+2\pi k\int_0^1\sin(2\pi kr)W(r)d(r)]$,

(L9): $\sqrt{T}\sum_{t=1}^{T}z_t\Delta\cos(2\pi kt/T)\xrightarrow{T}(2\pi k)^2\cdot\sigma[\int_0^1\cos(2\pi kr)W(r)d(r)]$,

(L10): $\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\sum_{s=1}^{t}z_s\Delta\sin(2\pi kt/T)$
$\xrightarrow{T}(2\pi k)\sigma\left(\int_0^1 W(s)ds+2\pi k\int_0^1\sin(2\pi kr)\left[\int_0^r W(s)ds\right]d(r)\right)$,

(L11): $\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\sum_{s=1}^{t}z_s\Delta\cos(2\pi kt/T)\xrightarrow{T}(2\pi k)^2\sigma\left(\int_0^1\cos(2\pi kr)\left[\int_0^r W(s)ds\right]d(r)\right)$.

Proof of Lemma 1: These results (L1)– (L6) are obtained from Enders and Lee (2009, p.43) and (L7) is obtained from Rodrigues and Taylor (2009, p. 24).

Proof of Lemma 2: (L8) and (L9) are given in Enders and Lee (2009, p.43). To prove (L8), we use the results in Bierens (1994, Lemma 9.6.3): $\sum F(t/T)x_t=F(1)S_T(1)-\int_0^1 f(r)S_T(r)d(r)$, where $S_T(r)=\sum_{t=1}^{[Tr]}x_t$ and $f(r)=F'(r)$. Letting $F(t/T)=\cos(2\pi kt/T)$, $x_t=\sum_{s=1}^{t}z_s$ and noting that $T^{-3/2}\sum_{t=1}^{[Tr]}x_t\to\sigma\int_0^r W(s)ds$ (see Hamilton, p.486), it is straightforward to obtain (L10). Similarly, the results in L(11) can be derived by setting $F(t/T)=\sin(2\pi kt/T)$.

Proof of Theorem 1:

[12] See for example, White (1999), Theorem 7.18.

Under the hypothesis of $\beta_i = 0$, the matrix format of (3) is

$$\begin{aligned} \Delta \mathbf{y}_i &= \alpha_{i,1}\boldsymbol{\Upsilon}_1 + \alpha_{i,2}\boldsymbol{\Upsilon}_2 + \gamma_i \mathbf{f} + \boldsymbol{\varepsilon}_i, & (33) \\ \mathbf{y}_{i,-1} &= y_{i,0}\boldsymbol{\tau} + \alpha_{i,1}\mathbf{s}_{\Upsilon_1,-1} + \alpha_{i,2}\mathbf{s}_{\Upsilon_2,-1} + \gamma_i \mathbf{s}_{f,-1} + \mathbf{s}_{i,-1}. & (34) \end{aligned}$$

Taking the cross-sectional average of (33) and (34), we obtain

$$\begin{aligned} \Delta \overline{\mathbf{y}} &= \overline{\alpha}_1\boldsymbol{\Upsilon}_1 + \overline{\alpha}_2\boldsymbol{\Upsilon}_2 + \overline{\gamma}\mathbf{f} + \overline{\boldsymbol{\varepsilon}}, & (35) \\ \overline{\mathbf{y}}_{-1} &= \overline{y_0}\boldsymbol{\tau} + \overline{\alpha}_1\mathbf{s}_{\Upsilon_1,-1} + \overline{\alpha}_2\mathbf{s}_{\Upsilon_2,-1} + \overline{\gamma}\mathbf{s}_{f,-1} + \overline{\mathbf{s}}_{-1}. & (36) \end{aligned}$$

Substituting (35) and (36) into (33) and (34), respectively, we have

$$\begin{aligned} \Delta \mathbf{y}_i &= \delta_i \Delta\overline{\mathbf{y}} + \zeta_{i,1}\boldsymbol{\Upsilon}_1 + \zeta_{i,2}\boldsymbol{\Upsilon}_2 + \boldsymbol{\xi}_i, & (37) \\ \mathbf{y}_{i,-1} &= (y_{i,0} - \delta_i\overline{y}_0)\boldsymbol{\tau} + \zeta_{i,1}\mathbf{s}_{\Upsilon_1,-1} + \zeta_{i,2}\mathbf{s}_{\Upsilon_2,-1} + \delta_i\overline{\mathbf{y}}_{-1} + \mathbf{s}_{i,-1} - \delta_i\overline{\mathbf{s}}_{-1}, & (38) \end{aligned}$$

where $\delta_i = \gamma_i/\overline{\gamma}$, $\zeta_{i,1} = \alpha_{i,1} - \delta_i\overline{\alpha}_1$, $\zeta_{i,2} = \alpha_{i,2} - \delta_i\overline{\alpha}_2$, $\boldsymbol{\xi}_i = \boldsymbol{\varepsilon}_i - \delta_i\overline{\boldsymbol{\varepsilon}} \sim (\mathbf{0}, \omega_i^2\mathbf{I}_T)$, $\overline{\mathbf{s}}_{-1} = \sum_{i=1}^N \mathbf{s}_{i,-1}$, $\overline{\gamma} = N^{-1}\sum_{i=1}^N \gamma_i$, $\overline{\alpha}_1 = N^{-1}\sum_{i=1}^N \alpha_{i,1}$, $\overline{\alpha}_2 = N^{-1}\sum_{i=1}^N \alpha_{i,2}$, $\overline{\boldsymbol{\varepsilon}} = N^{-1}\sum_{i=1}^N \boldsymbol{\varepsilon}_i$, and $\omega_i^2 = \sigma_i^2\left(1 - \frac{2\delta_i}{N}\right) + \frac{\delta_i^2}{N}\overline{\sigma}^2 = \sigma_i^2 + O\left(\frac{1}{N}\right)$, in which $\overline{\sigma}^2 = N^{-1}\sum_{i=1}^N \sigma_i^2 < \infty$. Besides, $\mathbf{M}_z\Delta\mathbf{y}_i = \omega_i\mathbf{M}_z\boldsymbol{\upsilon}_i$, $\mathbf{M}_z\mathbf{y}_{i,-1} = \omega_i\mathbf{M}_z\boldsymbol{s}_{i,-1}$ and[13] $\mathbf{M}_{i,z}\Delta\mathbf{y}_i = \omega_i\mathbf{M}_{i,z}\boldsymbol{\upsilon}_i$ in which $\boldsymbol{\upsilon}_i = \boldsymbol{\xi}_i/\omega_i \sim (\mathbf{0}, \mathbf{I}_T)$, $\boldsymbol{s}_{i,-1} = (\mathbf{s}_{i,-1} - \delta_i\overline{\mathbf{s}}_{-1})/\omega_i$, and $\boldsymbol{s}_{i,-1}$ is a random walk associated with υ. Using (7), we derive

$$t_i(N,T) = \frac{\frac{\boldsymbol{\upsilon}_i'\mathbf{M}_z\boldsymbol{s}_{i,-1}}{T}}{\left(\frac{\boldsymbol{\upsilon}_i'\mathbf{M}_{i,z}\boldsymbol{\upsilon}_i}{T-6}\right)^{1/2}\left(\frac{\boldsymbol{s}_{i,-1}'\mathbf{M}_z\boldsymbol{s}_{i,-1}}{T^2}\right)^{1/2}}. \tag{39}$$

We first consider the numerator of $t_i(N,T)$ and note that

$$\frac{\boldsymbol{\upsilon}_i'\mathbf{M}_z\boldsymbol{s}_{i,-1}}{T} = \frac{\boldsymbol{\upsilon}_i'\boldsymbol{s}_{i,-1}}{T} - (\boldsymbol{\upsilon}_i'\mathbf{Z}\mathbf{D})(\mathbf{D}\mathbf{Z}'\mathbf{Z}\mathbf{D})^{-1}\left(\frac{\mathbf{D}\mathbf{Z}'\boldsymbol{s}_{i,-1}}{T}\right),$$

where $\mathbf{D} = diag[T^{-1/2},\ T^{-1/2},\ T^{1/2},\ T^{1/2},\ T^{-1}]$. Also

[13] Here we have used the identity $\mathbf{M}_z\mathbf{s}_{\Upsilon_1,-1} = \mathbf{M}_z\boldsymbol{\Upsilon}_2 = \mathbf{0}$ and $\mathbf{M}_z\mathbf{s}_{\Upsilon_2,-1} = \mathbf{M}_z\boldsymbol{\Upsilon}_1 = \mathbf{0}$. From (L5) and (L6) in Lemma 1 and from the definition of $s_{\Upsilon_1,t} = \sum_{j=1}^t \Delta\sin(2\pi kj/T) = \sin(2\pi kt/T)$ and $s_{\Upsilon_2,t} = \sum_{j=1}^t \Delta\cos(2\pi kj/T) = \cos(2\pi kt/T)$, the results are obvious.

$$
\mathbf{DZ}'\boldsymbol{\upsilon}_i = \begin{bmatrix} \frac{\Delta\overline{\mathbf{y}}'\boldsymbol{\upsilon}_i}{\sqrt{T}} \\ \frac{\boldsymbol{\tau}'\boldsymbol{\upsilon}_i}{\sqrt{T}} \\ \sqrt{T}(\boldsymbol{\Upsilon}_1'\boldsymbol{\upsilon}_i) \\ \sqrt{T}(\boldsymbol{\Upsilon}_2'\boldsymbol{\upsilon}_i) \\ \frac{\overline{\mathbf{y}}_{-1}'\boldsymbol{\upsilon}_i}{T} \end{bmatrix}, \quad \frac{\mathbf{DZ}'\boldsymbol{s}_{i,-1}}{T} = \begin{bmatrix} \frac{\Delta\overline{\mathbf{y}}'\boldsymbol{s}_{i,-1}}{T^{3/2}} \\ \frac{\boldsymbol{\tau}'\boldsymbol{s}_{i,-1}}{T^{3/2}} \\ \frac{\boldsymbol{\Upsilon}_1'\boldsymbol{s}_{i,-1}}{T^{1/2}} \\ \frac{\boldsymbol{\Upsilon}_2'\boldsymbol{s}_{i,-1}}{T^{1/2}} \\ \frac{\overline{\mathbf{y}}_{-1}'\boldsymbol{s}_{i,-1}}{T^2}, \end{bmatrix}, \tag{40}
$$

$$
\mathbf{DZ}'\mathbf{ZD} = \begin{bmatrix} \frac{\Delta\overline{\mathbf{y}}'\Delta\overline{\mathbf{y}}}{T} & \frac{\Delta\overline{\mathbf{y}}'\boldsymbol{\tau}}{T} & \frac{\Delta\overline{\mathbf{y}}'\boldsymbol{\Upsilon}_1}{T^0} & \frac{\Delta\overline{\mathbf{y}}'\boldsymbol{\Upsilon}_2}{T^0} & \frac{\Delta\overline{\mathbf{y}}'\overline{\mathbf{y}}_{-1}}{T^{3/2}} \\ \frac{\boldsymbol{\tau}'\Delta\overline{\mathbf{y}}}{T} & 1 & \frac{\boldsymbol{\tau}'\boldsymbol{\Upsilon}_1}{T^0} & \frac{\boldsymbol{\tau}'\boldsymbol{\Upsilon}_2}{T^0} & \frac{\boldsymbol{\tau}'\overline{\mathbf{y}}_{-1}}{T^{3/2}} \\ \frac{\boldsymbol{\Upsilon}_1'\Delta\overline{\mathbf{y}}}{T^0} & \frac{\boldsymbol{\Upsilon}_1'\boldsymbol{\tau}}{T^0} & T(\boldsymbol{\Upsilon}_1'\boldsymbol{\Upsilon}_1) & T(\boldsymbol{\Upsilon}_1'\boldsymbol{\Upsilon}_2) & \frac{\boldsymbol{\Upsilon}_1'\overline{\mathbf{y}}_{-1}}{T^{1/2}} \\ \frac{\boldsymbol{\Upsilon}_2'\Delta\overline{\mathbf{y}}}{T^0} & \frac{\boldsymbol{\Upsilon}_2'\boldsymbol{\tau}}{T^0} & T(\boldsymbol{\Upsilon}_2'\boldsymbol{\Upsilon}_1) & T(\boldsymbol{\Upsilon}_2'\boldsymbol{\Upsilon}_2) & \frac{\boldsymbol{\Upsilon}_2'\overline{\mathbf{y}}_{-1}}{T^{1/2}} \\ \frac{\overline{\mathbf{y}}_{-1}'\Delta\overline{\mathbf{y}}}{T^{3/2}} & \frac{\overline{\mathbf{y}}_{-1}'\boldsymbol{\tau}}{T^{3/2}} & \frac{\overline{\mathbf{y}}_{-1}'\boldsymbol{\Upsilon}_1}{T^{1/2}} & \frac{\overline{\mathbf{y}}_{-1}'\boldsymbol{\Upsilon}_2}{T^{1/2}} & \frac{\overline{\mathbf{y}}_{-1}'\overline{\mathbf{y}}_{-1}}{T^2} \end{bmatrix}. \tag{41}
$$

Based on (35), (36), $\boldsymbol{\upsilon}_i = (\boldsymbol{\varepsilon}_i - \delta_i\overline{\boldsymbol{\varepsilon}})/\omega_i$, $\boldsymbol{s}_{i,-1} = (\mathbf{s}_{i,-1} - \delta_i\overline{\mathbf{s}}_{-1})/\omega_i$ and the results in Pesaran (2007, section A.2), we simplify the elements in the numerator of $t_i(N,T)$ with the following expression:

$$
\begin{aligned}
\frac{\boldsymbol{\upsilon}_i'\boldsymbol{s}_{i,-1}}{T} &= \frac{(\boldsymbol{\varepsilon}_i'\mathbf{s}_{i,-1} - \delta_i\boldsymbol{\varepsilon}_i'\overline{\mathbf{s}}_{-1} - \delta_i\overline{\boldsymbol{\varepsilon}}'\mathbf{s}_{i,-1} + \delta_i^2\overline{\boldsymbol{\varepsilon}}'\overline{\mathbf{s}}_{-1})}{T\omega_i^2}, \\
\sqrt{T}(\boldsymbol{\Upsilon}_1'\boldsymbol{\upsilon}_i) &= \frac{\sqrt{T}\left(\boldsymbol{\Upsilon}_1'\boldsymbol{\varepsilon}_i - \delta_i(\boldsymbol{\Upsilon}_1'\overline{\boldsymbol{\varepsilon}})\right)}{\omega_i}, \quad \sqrt{T}(\boldsymbol{\Upsilon}_2'\boldsymbol{\upsilon}_i) = \frac{\sqrt{T}\left(\boldsymbol{\Upsilon}_2'\boldsymbol{\varepsilon}_i - \delta_i(\boldsymbol{\Upsilon}_2'\overline{\boldsymbol{\varepsilon}})\right)}{\omega_i}, \\
\frac{\boldsymbol{\tau}'\boldsymbol{\upsilon}_i}{\sqrt{T}} &= \frac{\boldsymbol{\tau}'\boldsymbol{\varepsilon}_i - \delta_i(\boldsymbol{\tau}'\overline{\boldsymbol{\varepsilon}})}{\omega_i\sqrt{T}}, \\
\frac{\Delta\overline{\mathbf{y}}'\boldsymbol{\upsilon}_i}{\sqrt{T}} &= \overline{\alpha}_1\left(\frac{\boldsymbol{\Upsilon}_1'\boldsymbol{\varepsilon}_i - \delta_i(\boldsymbol{\Upsilon}_1'\overline{\boldsymbol{\varepsilon}})}{\omega_i\sqrt{T}}\right) + \overline{\alpha}_2\left(\frac{\boldsymbol{\Upsilon}_2'\boldsymbol{\varepsilon}_i - \delta_i(\boldsymbol{\Upsilon}_2'\overline{\boldsymbol{\varepsilon}})}{\omega_i\sqrt{T}}\right) \\
&\quad + \overline{\gamma}\left(\frac{\mathbf{f}'\boldsymbol{\varepsilon}_i - \delta_i(\mathbf{f}'\overline{\boldsymbol{\varepsilon}})}{\omega_i\sqrt{T}}\right) + \left(\frac{\overline{\boldsymbol{\varepsilon}}'\boldsymbol{\varepsilon}_i - \delta_i(\overline{\boldsymbol{\varepsilon}}'\overline{\boldsymbol{\varepsilon}})}{\omega_i\sqrt{T}}\right), \\
\frac{\overline{\mathbf{y}}_{-1}'\boldsymbol{\upsilon}_i}{T} &= \overline{y}_0\left(\frac{\boldsymbol{\tau}'\boldsymbol{\varepsilon}_i - \delta_i(\boldsymbol{\tau}'\overline{\boldsymbol{\varepsilon}})}{\omega_i T}\right) + \overline{\alpha}_1\left(\frac{\mathbf{s}_{\Upsilon_1,-1}'\boldsymbol{\varepsilon}_i - \delta_i(\mathbf{s}_{\Upsilon_1,-1}'\overline{\boldsymbol{\varepsilon}})}{\omega_i T}\right) \\
&\quad + \overline{\alpha}_2\left(\frac{\mathbf{s}_{\Upsilon_2,-1}'\boldsymbol{\varepsilon}_i - \delta_i(\mathbf{s}_{\Upsilon_2,-1}'\overline{\boldsymbol{\varepsilon}})}{\omega_i T}\right) + \overline{\gamma}\left(\frac{\mathbf{s}_{f,-1}'\boldsymbol{\varepsilon}_i - \delta_i(\mathbf{s}_{f,-1}'\overline{\boldsymbol{\varepsilon}})}{\omega_i T}\right) \\
&\quad + \frac{\overline{\mathbf{s}}_{-1}'\boldsymbol{\varepsilon}_i - \delta_i(\overline{\mathbf{s}}_{-1}'\overline{\boldsymbol{\varepsilon}})}{\omega_i T}, \\
\frac{\boldsymbol{\Upsilon}_1'\boldsymbol{s}_{i,-1}}{\sqrt{T}} &= \frac{\boldsymbol{\Upsilon}_1'\mathbf{s}_{i,-1} - \delta_i(\boldsymbol{\Upsilon}_1'\overline{\mathbf{s}}_{-1})}{\omega_i\sqrt{T}}, \quad \frac{\boldsymbol{\Upsilon}_2'\boldsymbol{s}_{i,-1}}{\sqrt{T}} = \frac{\boldsymbol{\Upsilon}_2'\mathbf{s}_{i,-1} - \delta_i(\boldsymbol{\Upsilon}_2'\overline{\mathbf{s}}_{-1})}{\omega_i\sqrt{T}}, \\
\frac{\Delta\overline{\mathbf{y}}'\boldsymbol{s}_{i,-1}}{T^{3/2}} &= \overline{\alpha}_1\left(\frac{\boldsymbol{\Upsilon}_1'\mathbf{s}_{i,-1} - \delta_i(\boldsymbol{\Upsilon}_1'\overline{\mathbf{s}}_{-1})}{\omega_i T^{3/2}}\right) + \overline{\alpha}_2\left(\frac{\boldsymbol{\Upsilon}_2'\mathbf{s}_{i,-1} - \delta_i(\boldsymbol{\Upsilon}_2'\overline{\mathbf{s}}_{-1})}{\omega_i T^{3/2}}\right) \\
&\quad + \overline{\gamma}\left(\frac{\mathbf{f}'\mathbf{s}_{i,-1} - \delta_i(\mathbf{f}'\overline{\mathbf{s}}_{\mathbf{-1}})}{\omega_i T^{3/2}}\right) + \left(\frac{\overline{\boldsymbol{\varepsilon}}'\mathbf{s}_{i,-1} - \delta_i(\overline{\boldsymbol{\varepsilon}}'\overline{\mathbf{s}}_{-1})}{\omega_i T^{3/2}}\right),
\end{aligned}
$$

$$
\begin{aligned}
\frac{\boldsymbol{\tau}'\boldsymbol{s}_{i,-1}}{T^{3/2}} &= \frac{\boldsymbol{\tau}'\mathbf{s}_{i,-1}-\delta_i(\boldsymbol{\tau}'\overline{\mathbf{s}}_{-1})}{\omega_i T^{3/2}},\\
\frac{\overline{\mathbf{y}}'_{-1}\boldsymbol{s}_{i,-1}}{T^2} &= \overline{y}_0\left(\frac{\boldsymbol{\tau}'\mathbf{s}_{i,-1}-\delta_i(\boldsymbol{\tau}'\overline{\mathbf{s}}_{-1})}{\omega_i T^2}\right)+\overline{\alpha}_1\left(\frac{\mathbf{s}'_{\Upsilon_1,-1}\mathbf{s}_{i,-1}-\delta_i(\mathbf{s}'_{\Upsilon_1,-1}\overline{\mathbf{s}}_{-1})}{\omega_i T^2}\right)\\
&\quad+\overline{\alpha}_2\left(\frac{\mathbf{s}'_{\Upsilon_2,-1}\mathbf{s}_{i,-1}-\delta_i(\mathbf{s}'_{\Upsilon_2,-1}\overline{\mathbf{s}}_{-1})}{\omega_i T^2}\right)\\
&\quad+\overline{\gamma}\left(\frac{\mathbf{s}'_{f,-1}\mathbf{s}_{i,-1}-\delta_i(\mathbf{s}'_{f,-1}\overline{\mathbf{s}}_{-1})}{\omega_i T^2}\right)+\frac{\overline{\mathbf{s}}'_{-1}\mathbf{s}_{i,-1}-\delta_i(\overline{\mathbf{s}}'_{-1}\overline{\mathbf{s}}_{-1})}{\omega_i T^2},\\
\frac{\boldsymbol{\Upsilon}'_1\Delta\overline{\mathbf{y}}}{T^0} &= \overline{\alpha}_1\boldsymbol{\Upsilon}'_1\boldsymbol{\Upsilon}_1+\overline{\alpha}_2\boldsymbol{\Upsilon}'_1\boldsymbol{\Upsilon}_2+\overline{\gamma}\boldsymbol{\Upsilon}'_1\mathbf{f}+\boldsymbol{\Upsilon}'_1\overline{\boldsymbol{\varepsilon}},\\
\frac{\boldsymbol{\Upsilon}'_1\overline{\mathbf{y}}_{-1}}{T^{1/2}} &= T^{-1/2}(\overline{y_0}\boldsymbol{\Upsilon}'_1\boldsymbol{\tau}+\overline{\alpha}_1\boldsymbol{\Upsilon}'_1\mathbf{s}_{\Upsilon_1,-1}+\overline{\alpha}_2\boldsymbol{\Upsilon}'_1\mathbf{s}_{\Upsilon_2,-1}+\overline{\gamma}\boldsymbol{\Upsilon}'_1\mathbf{s}_{f,-1}+\boldsymbol{\Upsilon}'_1\overline{\mathbf{s}}_{-1}),\\
\frac{\boldsymbol{\Upsilon}'_2\Delta\overline{\mathbf{y}}}{T^0} &= \overline{\alpha}_1\boldsymbol{\Upsilon}'_2\boldsymbol{\Upsilon}_1+\overline{\alpha}_2\boldsymbol{\Upsilon}'_2\boldsymbol{\Upsilon}_2+\overline{\gamma}\boldsymbol{\Upsilon}'_2\mathbf{f}+\boldsymbol{\Upsilon}'_2\overline{\boldsymbol{\varepsilon}},\\
\frac{\boldsymbol{\Upsilon}'_2\overline{\mathbf{y}}_{-1}}{T^{1/2}} &= T^{-1/2}(\overline{y_0}\boldsymbol{\Upsilon}'_2\boldsymbol{\tau}+\overline{\alpha}_1\boldsymbol{\Upsilon}'_2\mathbf{s}_{\Upsilon_1,-1}+\overline{\alpha}_2\boldsymbol{\Upsilon}'_2\mathbf{s}_{\Upsilon_2,-1}+\overline{\gamma}\boldsymbol{\Upsilon}'_2\mathbf{s}_{f,-1}+\boldsymbol{\Upsilon}'_2\overline{\mathbf{s}}_{-1}),\\
\frac{\Delta\overline{\mathbf{y}}'\Delta\overline{\mathbf{y}}}{T} &= T^{-1}(\overline{\alpha}_1^2\boldsymbol{\Upsilon}'_1\boldsymbol{\Upsilon}_1+\overline{\alpha}_2^2\boldsymbol{\Upsilon}'_2\boldsymbol{\Upsilon}_2+\overline{\gamma}^2\mathbf{f}'\mathbf{f}+\overline{\boldsymbol{\varepsilon}}'\overline{\boldsymbol{\varepsilon}}+2\overline{\alpha}_1\overline{\alpha}_2\boldsymbol{\Upsilon}'_1\boldsymbol{\Upsilon}_2+2\overline{\alpha}_1\overline{\gamma}\boldsymbol{\Upsilon}'_1\mathbf{f}\\
&\quad+2\overline{\alpha}_1\boldsymbol{\Upsilon}'_1\overline{\boldsymbol{\varepsilon}}+2\overline{\alpha}_2\overline{\gamma}\boldsymbol{\Upsilon}'_2\mathbf{f}+2\overline{\alpha}_2\boldsymbol{\Upsilon}'_2\overline{\boldsymbol{\varepsilon}}+2\overline{\gamma}\mathbf{f}'\overline{\boldsymbol{\varepsilon}}),\\
\frac{\Delta\overline{\mathbf{y}}'\boldsymbol{\tau}}{T} &= T^{-1}(\overline{\alpha}_1\boldsymbol{\Upsilon}'_1\boldsymbol{\tau}+\overline{\alpha}_2\boldsymbol{\Upsilon}'_2\boldsymbol{\tau}+\overline{\gamma}\mathbf{f}'\boldsymbol{\tau}+\overline{\boldsymbol{\varepsilon}}'\boldsymbol{\tau}),\\
\frac{\boldsymbol{\tau}'\overline{\mathbf{y}}_{-1}}{T^{3/2}} &= T^{-3/2}(\overline{y_0}\boldsymbol{\tau}'\boldsymbol{\tau}+\overline{\alpha}_1\boldsymbol{\tau}'\mathbf{s}_{\Upsilon_1,-1}+\overline{\alpha}_2\boldsymbol{\tau}'\mathbf{s}_{\Upsilon_2,-1}+\overline{\gamma}\boldsymbol{\tau}'\mathbf{s}_{f,-1}+\boldsymbol{\tau}'\overline{\mathbf{s}}_{-1}),\\
\frac{\Delta\overline{\mathbf{y}}'\overline{\mathbf{y}}_{-1}}{T^{3/2}} &= T^{-3/2}[(\overline{\alpha}_1\overline{y_0}\boldsymbol{\Upsilon}'_1\boldsymbol{\tau}+\overline{\alpha}_1^2\boldsymbol{\Upsilon}'_1\mathbf{s}_{\Upsilon_1,-1}+\overline{\alpha}_1\overline{\alpha}_2\boldsymbol{\Upsilon}'_1\mathbf{s}_{\Upsilon_2,-1}+\\
&\quad\overline{\alpha}_1\overline{\gamma}\boldsymbol{\Upsilon}'_1\mathbf{s}_{f,-1}+\overline{\alpha}_1\boldsymbol{\Upsilon}'_1\overline{\mathbf{s}}_{-1})+(\overline{\alpha}_2\overline{y_0}\boldsymbol{\Upsilon}'_2\boldsymbol{\tau}+\overline{\alpha}_1\overline{\alpha}_2\boldsymbol{\Upsilon}'_2\mathbf{s}_{\Upsilon_1,-1}\\
&\quad+\overline{\alpha}_2^2\boldsymbol{\Upsilon}'_2\mathbf{s}_{\Upsilon_2,-1}+\overline{\alpha}_2\overline{\gamma}\boldsymbol{\Upsilon}'_2\mathbf{s}_{f,-1}+\overline{\alpha}_2\boldsymbol{\Upsilon}'_2\overline{\mathbf{s}}_{-1})+(\overline{\gamma y_0}\mathbf{f}'\boldsymbol{\tau}+\overline{\alpha}_1\overline{\gamma}\mathbf{f}'\mathbf{s}_{\Upsilon_1,-1}\\
&\quad+\overline{\alpha}_2\overline{\gamma}\mathbf{f}'\mathbf{s}_{\Upsilon_2,-1}+\overline{\gamma}^2\mathbf{f}'\mathbf{s}_{f,-1}+\overline{\gamma}\mathbf{f}'\mathbf{s}_{f,-1})+(\overline{y_0}\overline{\boldsymbol{\varepsilon}}'\boldsymbol{\tau}+\overline{\alpha}_1\overline{\boldsymbol{\varepsilon}}'\mathbf{s}_{\Upsilon_1,-1}\\
&\quad+\overline{\alpha}_2\overline{\boldsymbol{\varepsilon}}'\mathbf{s}_{\Upsilon_2,-1}+\overline{\gamma}\overline{\boldsymbol{\varepsilon}}'\mathbf{s}_{f,-1}+\overline{\boldsymbol{\varepsilon}}'\overline{\mathbf{s}}_{-1})],\\
\frac{\overline{\mathbf{y}}'_{-1}\overline{\mathbf{y}}_{-1}}{T^2} &= T^{-2}(\overline{y}_0^2\boldsymbol{\tau}'\boldsymbol{\tau}+\overline{\alpha}_1^2\mathbf{s}'_{\Upsilon_1,-1}\mathbf{s}_{\Upsilon_1,-1}+\overline{\alpha}_2^2\mathbf{s}'_{\Upsilon_2,-1}\mathbf{s}_{\Upsilon_2,-1}+\overline{\gamma}^2\mathbf{s}'_{f,-1}\mathbf{s}_{f,-1}\\
&\quad+\overline{\mathbf{s}}'_{-1}\overline{\mathbf{s}}_{-1}+2\overline{y}_0\overline{\alpha}_1\boldsymbol{\tau}'\mathbf{s}_{\Upsilon_1,-1}+2\overline{y}_0\overline{\alpha}_2\boldsymbol{\tau}'\mathbf{s}_{\Upsilon_2,-1}+2\overline{y}_0\overline{\gamma}\boldsymbol{\tau}'\mathbf{s}_{f,-1}\\
&\quad+2\overline{y}_0\boldsymbol{\tau}'\overline{\mathbf{s}}_{-1}+2\overline{\alpha}_1\overline{\alpha}_2\mathbf{s}'_{\Upsilon_1,-1}\mathbf{s}_{\Upsilon_2,-1}+2\overline{\alpha}_1\overline{\gamma}\mathbf{s}_{\Upsilon_1,-1}\mathbf{s}_{f,-1}\\
&\quad+2\overline{\alpha}_1\mathbf{s}_{\Upsilon_1,-1}\overline{\mathbf{s}}_{-1}+2\overline{\alpha}_2\overline{\gamma}\mathbf{s}'_{\Upsilon_2,-1}\mathbf{s}_{f,-1}+2\overline{\alpha}_2\mathbf{s}'_{\Upsilon_2,-1}\overline{\mathbf{s}}_{-1}+2\overline{\gamma}\mathbf{s}'_{f,-1}\overline{\mathbf{s}}_{-1}).
\end{aligned}
$$

As in Pesaran (2007), since all stochastic terms involving averaging over i go to zero as N goes to infinity, we therefore obtain

$$
\mathbf{DZ}'\boldsymbol{v}_i \xrightarrow{N} \boldsymbol{\Gamma}_T\mathbf{q}_{iT},\ \frac{\mathbf{DZ}'\boldsymbol{s}_{i,-1}}{T} \xrightarrow{N} \boldsymbol{\Gamma}_T\mathbf{h}_{iT},\ \ \mathbf{DZ}'\mathbf{ZD} \xrightarrow{N} \boldsymbol{\Gamma}_T\boldsymbol{\Psi}_{fT}\boldsymbol{\Gamma}'_T, \tag{42}
$$

where $\mathbf{\Gamma}_T,\ \mathbf{q}_{iT},\ \mathbf{h}_{iT}$ and $\mathbf{\Psi}_{fT}$ are defined in (11), (12) and (14).

Similarly, the first element in the denominator of the t-statistic in (39) can be expressed as

$$\begin{aligned}
\frac{\boldsymbol{\upsilon}_i'\mathbf{M}_{i,z}\boldsymbol{\upsilon}_i}{T-6} &= \frac{\boldsymbol{\upsilon}_i'\boldsymbol{\upsilon}_i}{T-6} - \frac{(\boldsymbol{\upsilon}_i'\mathbf{G}_i\mathbf{E})(\mathbf{E}\mathbf{G}_i'\mathbf{G}_i\mathbf{E})^{-1}(\mathbf{E}\mathbf{G}_i'\boldsymbol{\upsilon}_i)}{T-6} \\
&\xrightarrow{N} \frac{\boldsymbol{\varepsilon}_i'\boldsymbol{\varepsilon}_i}{\sigma_i^2(T-6)} - \frac{(\mathbf{d}_{iT}'\mathbf{\Theta}_T')(\mathbf{\Theta}_T\mathbf{\Xi}_{iT}\mathbf{\Theta}_T')^{-1}(\mathbf{\Theta}_T\mathbf{d}_{iT})}{T-6},
\end{aligned} \tag{43}$$

where $\mathbf{E} = diag[T^{-1/2},\ T^{-1/2},\ T^{1/2},\ T^{1/2},\ T^{-1},\ (\omega_i T)^{-1}]$, $\mathbf{d}_{iT},\ \mathbf{\Xi}_{iT}$ and $\mathbf{\Theta}_T$ are defined in (12)-(14). Substituting (42) and (43) into (39), we obtain

$$t_i(N,T) \xrightarrow{N} \frac{\frac{\boldsymbol{\varepsilon}_i'\mathbf{s}_{i,-1}}{\sigma_i^2 T} - (\mathbf{q}_{iT}'\mathbf{\Gamma}_T')(\mathbf{\Gamma}_T\mathbf{\Psi}_{f,T}\mathbf{\Gamma}_T')^{-1}(\mathbf{\Gamma}_T\mathbf{h}_{iT})}{J_1 \times J_2}.$$

Obviously, the asymptotic distribution of $t_i(N,T)$ is not free of nuisance parameters as $N \to \infty$ since $\mathbf{\Theta}_T$ and $\mathbf{\Gamma}_T$ are not diagonal matrices.

Proof of Theorem 2:

Given the results from Theorem 1, we now allow T to go to infinity. Using Lemmas 1 and 2 as well as the results in Hamilton (1994, p.486) and Pesaran (2007, Section A.2.2), the following results can be straightforwardly derived. The convergence results for the numerator of the t-statistic are:

$$\begin{aligned}
T\mathbf{\Upsilon}_1'\mathbf{\Upsilon}_1 &\xrightarrow{T} (2\pi k)^2\int_0^1 \cos^2(2\pi kr)dr,\ T\mathbf{\Upsilon}_2'\mathbf{\Upsilon}_2 \xrightarrow{T} (2\pi k)^2\int_0^1 \sin^2(2\pi kr)dr, \\
\frac{\sqrt{T}(\mathbf{\Upsilon}_1'\boldsymbol{\varepsilon}_i)}{\sigma_i} &\xrightarrow{T} (2\pi k)[W(1) + 2\pi k\int_0^1 \sin(2\pi kr)W(r)d(r)], \\
\frac{\sqrt{T}(\mathbf{\Upsilon}_2'\boldsymbol{\varepsilon}_i)}{\sigma_i} &\xrightarrow{T} (2\pi k)^2\cdot\left[\int_0^1 \cos(2\pi kr)W(r)d(r)\right], \\
\frac{\mathbf{\Upsilon}_1'\mathbf{s}_{i,-1}}{\sigma_i\sqrt{T}} &\xrightarrow{T} (2\pi k)\left(\int_0^1 W(s)ds + 2\pi k\int_0^1 \sin(2\pi kr)\left[\int_0^r W(s)ds\right]d(r)\right), \\
\frac{\mathbf{\Upsilon}_2'\mathbf{s}_{i,-1}}{\sigma_i\sqrt{T}} &\xrightarrow{T} (2\pi k)^2\left(\int_0^1 \cos(2\pi kr)\left[\int_0^r W(s)ds\right]d(r)\right), \\
\frac{\mathbf{\Upsilon}_1'\mathbf{s}_{f,-1}}{T^{1/2}} &\xrightarrow{T} (2\pi k)\left(\int_0^1 W_f(s)ds + 2\pi k\int_0^1 \sin(2\pi kr)\left[\int_0^r W_f(s)ds\right]d(r)\right), \\
\frac{\mathbf{\Upsilon}_2'\mathbf{s}_{f,-1}}{T^{1/2}} &\xrightarrow{T} (2\pi k)^2\left(\int_0^1 \cos(2\pi kr)\left[\int_0^r W_f(s)ds\right]d(r)\right), \\
T\mathbf{\Upsilon}_1'\mathbf{\Upsilon}_2 &\xrightarrow{T} 0,\ \mathbf{\Upsilon}_1'\mathbf{f} \xrightarrow{T} 0,\ \mathbf{\Upsilon}_2'\mathbf{f} \xrightarrow{T} 0,\ \mathbf{\Upsilon}_1'\boldsymbol{\tau} \xrightarrow{T} 0,\ \mathbf{\Upsilon}_2'\boldsymbol{\tau} \xrightarrow{T} 0, \\
T^{-1}\boldsymbol{\tau}'\mathbf{f} &\xrightarrow{T} 0\ ,\ \frac{\mathbf{f}'\boldsymbol{\varepsilon}_i}{\sigma_i\sqrt{T}} \xrightarrow{T} W_{fi}(1),\ \frac{\boldsymbol{\tau}'\boldsymbol{\varepsilon}_i}{\sigma_i\sqrt{T}} \xrightarrow{T} W_i(1),
\end{aligned}$$

$$
\begin{aligned}
\frac{\mathbf{s}'_{f,-1}\boldsymbol{\varepsilon}_i}{\sigma_i T} &\xrightarrow{T} \int_0^1 W_f(r)dW_i(r),\ \frac{\mathbf{f}'\mathbf{f}}{T} \to 1, \frac{\mathbf{s}'_{i,-1}\boldsymbol{\varepsilon}_i}{\sigma_i^2 T} \xrightarrow{T} \int_0^1 W_i(r)dW_i(r),\\
\frac{\mathbf{f}'\mathbf{s}_{i,-1}}{\sigma_i T^{3/2}} &\xrightarrow{T} 0,\ \frac{\boldsymbol{\tau}'\mathbf{s}_{i,-1}}{\sigma_i T^{3/2}} \xrightarrow{T} \int_0^1 W_i(r)dr,\ \frac{\mathbf{f}'\mathbf{s}_{f,-1}}{T^{3/2}} \xrightarrow{T} 0,\\
\frac{\boldsymbol{\tau}'\mathbf{s}_{f,-1}}{T^{3/2}} &\xrightarrow{T} \int_0^1 W_f(r)dr,\ \frac{\mathbf{s}'_{f,-1}\mathbf{s}_{i,-1}}{\sigma_i T^2} \xrightarrow{T} \int_0^1 W_f(r)W_i(r)dr,\\
\frac{\mathbf{s}'_{f,-1}\mathbf{s}_{f,-1}}{T^2} &\xrightarrow{T} \int_0^1 W_f^2(r)dr,
\end{aligned}
$$

where $W_i(r)$ and $W_f(r)$ are independently distributed Brownian motions defined on $[0,1]$. Using the above results, it would be obvious that

$$
\boldsymbol{\Gamma}_T\mathbf{q}_{iT} \xrightarrow{T} \boldsymbol{\Gamma}^*\mathbf{q}^*_{if},\ \ \boldsymbol{\Gamma}_T\mathbf{h}_{iT} \xrightarrow{T} \boldsymbol{\Gamma}^*\mathbf{h}^*_{if},\ \ \boldsymbol{\Gamma}_T\boldsymbol{\Psi}_{fT}\boldsymbol{\Gamma}'_T \xrightarrow{T} \boldsymbol{\Gamma}^*\boldsymbol{\Psi}^*_f\boldsymbol{\Gamma}'^*,
$$

where $\boldsymbol{\Gamma}^* = diag[\gamma^*,\ 1,\ 1,\ 1,\ \gamma^*]$ and

$$
\begin{aligned}
\mathbf{q}^*_{if} &= \begin{bmatrix} W_{fi}(1) \\ W_i(1) \\ (2\pi k)\sigma[W(1) + 2\pi k\int_0^1 \sin(2\pi kr)W(r)d(r)] \\ (2\pi k)^2\cdot\sigma[\int_0^1 \cos(2\pi kr)W(r)d(r)] \\ \int_0^1 W_f(r)dW_i(r) \end{bmatrix} = \begin{bmatrix} W_{fi}(1) \\ \mathbf{q}_{if} \end{bmatrix},\\
\mathbf{h}^*_{if} &= \begin{bmatrix} 0 \\ \int_0^1 W_i(r)dr \\ (2\pi k)\left(\int_0^1 W(s)ds + 2\pi k\int_0^1 \sin(2\pi kr)\left[\int_0^r W(s)ds\right]d(r)\right) \\ (2\pi k)^2\left(\int_0^1 \cos(2\pi kr)\left[\int_0^r W(s)ds\right]d(r)\right) \\ \int_0^1 W_f(r)W_i(r)dr \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{h}_{if} \end{bmatrix},\\
\boldsymbol{\Psi}^*_f &= \begin{bmatrix} 1_{1\times1} & \mathbf{0}'_{1\times4} \\ \mathbf{0}_{4\times1} & \boldsymbol{\Psi}_{f(4\times4)} \end{bmatrix},\ \ \boldsymbol{\Psi}_{f(4\times4)} = \begin{bmatrix} \mathbf{A}_{2\times2} & \mathbf{B} \\ \mathbf{B}' & \mathbf{D}_{2\times2} \end{bmatrix} \text{is as defined in (18).}
\end{aligned}
$$

Therefore, the numerator of the t-statistic is distributed as

$$
\int_0^1 W_i(r)dW_i(r) - \mathbf{q}'^*_{if}\boldsymbol{\Psi}^{*\,-1}_f\mathbf{h}^*_{if} = \int_0^1 W_i(r)dW_i(r) - \mathbf{q}'_{if}\boldsymbol{\Psi}^{-1}_f\mathbf{h}_{if}. \tag{44}
$$

Similarly, the convergence results for the denominator of the t-statistic are:

$$
\frac{\mathbf{s}'_{i,-1}\mathbf{s}_{i,-1}}{\sigma_i^2 T^2} \xrightarrow{T} \int_0^1 W_i^2(r)d(r),\ \ \frac{(\mathbf{d}'_{iT}\boldsymbol{\Theta}'_T)(\boldsymbol{\Theta}_T\boldsymbol{\Xi}_{iT}\boldsymbol{\Theta}'_T)^{-1}(\boldsymbol{\Theta}_T\mathbf{d}_{iT})}{T-6} \xrightarrow{T} 0,
$$

$$\frac{\varepsilon_i'\varepsilon_i}{\sigma_i^2(T-6)} \xrightarrow{T} 1,\ (\mathbf{h}_{i,T}'\mathbf{\Gamma}_T')(\mathbf{\Gamma}_T\mathbf{\Psi}_{fT}\mathbf{\Gamma}_T')^{-1}(\mathbf{\Gamma}_T\mathbf{h}_{iT}) \xrightarrow{T} \mathbf{h}_{if}'\mathbf{\Psi}_f^{-1}\mathbf{h}_{if}. \tag{45}$$

The joint convergence results in (44) and (45) together with the application of the continuous mapping theorem are sufficient to establish the stated result in Theorem 2.

Proof of Corollary:

Given Pesaran's (2007) $CADF$ regression: $\Delta y_{it} = c_{i,0} + c_{i,3}\overline{y}_{t-1} + c_{i,4}\Delta\overline{y}_t + b_i y_{i,t-1} + e_{it}$, the t-statistic of b_i used to test the unit root hypothesis can be expressed as

$$t_i^{P,B}(N,T) = \frac{\Delta\mathbf{y}_i'\mathbf{M}_p\mathbf{y}_{i,-1}}{\hat{\sigma}_i(\mathbf{y}_{i,-1}'\mathbf{M}_p\mathbf{y}_{i,-1})^{1/2}}, \tag{46}$$

where $\hat{\sigma}_i^2 = \frac{\Delta\mathbf{y}_i'\mathbf{M}_{i,p}\Delta\mathbf{y}_i}{T-4}$, $\mathbf{M}_p = \mathbf{I}_T - \mathbf{P}(\mathbf{P}'\mathbf{P})^{-1}\mathbf{P}'$, $\mathbf{M}_{i,p} = \mathbf{I}_T - \mathbf{F}_i(\mathbf{F}_i'\mathbf{F}_i)^{-1}\mathbf{F}_i'$, $\mathbf{F}_i = (\mathbf{P}, \mathbf{y}_{i,-1})$, $\mathbf{P} = (\Delta\overline{\mathbf{y}}, \boldsymbol{\tau}, \overline{\mathbf{y}}_{-1})$. Assuming that y_{it} is generated based on (3) under the null hypothesis in (4), we then substitute (37) and (38) into (46) to obtain

$$t_i^{P,B}(N,T) \quad = \quad \frac{\frac{\boldsymbol{v}_i'\mathbf{M}_p\boldsymbol{s}_{i,-1}}{T}}{\left(\frac{\boldsymbol{v}_i'\mathbf{M}_{i,p}\boldsymbol{v}_i}{T-4}\right)^{1/2}\left(\frac{\boldsymbol{s}_{i,-1}'\mathbf{M}_p\boldsymbol{s}_{i,-1}}{T^2}\right)^{1/2}} \oplus \frac{\aleph_{i,T}}{\hbar_{i,T}}, \tag{47}$$

where

$$\begin{aligned}
\aleph_{i,T} \quad &= \quad \frac{\left(\mathbf{\Upsilon}'^*\mathbf{M}_p\boldsymbol{s}_{i,-1} + \boldsymbol{v}_i'\mathbf{M}_p\mathbf{s}_{\Upsilon_{-1}^*} + \omega_i^{-1}\mathbf{\Upsilon}'^*\mathbf{M}_p\mathbf{s}_{\Upsilon_{-1}^*}\right)}{\omega_i T}, \\
\hbar_{i,T} \quad &= \quad \left(\frac{\boldsymbol{v}_i'\mathbf{M}_{i,p}\boldsymbol{v}_i}{T}\right)^{1/2}\left(\frac{\left(\boldsymbol{s}_{i,-1}'\mathbf{M}_p\mathbf{s}_{\Upsilon_{-1}^*} + \mathbf{s}_{\Upsilon_{-1}^*}'\mathbf{M}_p\boldsymbol{s}_{i,-1} + \omega_i^{-1}\mathbf{s}_{\Upsilon_{-1}^*}'\mathbf{M}_p\mathbf{s}_{\Upsilon_{-1}^*}\right)}{\omega_i T^2}\right)^{1/2} \\
&\quad + \left(\frac{\left(\boldsymbol{v}_i'\mathbf{M}_{i,p}\mathbf{\Upsilon}^* + \mathbf{\Upsilon}'^*\mathbf{M}_{i,p}\boldsymbol{v}_i + \omega_i^{-1}\mathbf{\Upsilon}'^*\mathbf{M}_{i,p}\mathbf{\Upsilon}^*\right)}{\omega_i T}\right)^{1/2}\left(\frac{\boldsymbol{s}_{i,-1}'\mathbf{M}_p\boldsymbol{s}_{i,-1}}{T^2}\right)^{1/2} \\
&\quad + \left[\left(\frac{\left(\boldsymbol{v}_i'\mathbf{M}_{i,p}\mathbf{\Upsilon}^* + \mathbf{\Upsilon}'^*\mathbf{M}_{i,p}\boldsymbol{v}_i + \omega_i^{-1}\mathbf{\Upsilon}'^*\mathbf{M}_{i,p}\mathbf{\Upsilon}^*\right)}{\omega_i T}\right)^{1/2} \times \right. \\
&\quad \left.\left(\frac{\left(\boldsymbol{s}_{i,-1}'\mathbf{M}_p\mathbf{s}_{\Upsilon_{-1}^*} + \mathbf{s}_{\Upsilon_{-1}^*}'\mathbf{M}_p\boldsymbol{s}_{i,-1} + \omega_i^{-1}\mathbf{s}_{\Upsilon_{-1}^*}'\mathbf{M}_p\mathbf{s}_{\Upsilon_{-1}^*}\right)}{\omega_i T^2}\right)^{1/2}\right],
\end{aligned}$$

$\mathbf{\Upsilon}^* = \zeta_{i,1}\mathbf{\Upsilon}_1 + \zeta_{i,2}\mathbf{\Upsilon}_2$ and $\mathbf{s}_{\Upsilon_{-1}^*} = \zeta_{i,1}\mathbf{s}_{\Upsilon_{1},-1} + \zeta_{i,2}\mathbf{s}_{\Upsilon_{2},-1}$. Here $\aleph_{i,T}$ is the bias of slope estimation and $\hbar_{i,T}$ is the bias of standard error estimation resulting from omitting breaks in Pesaran's $CADF$ regression. The notation "$\oplus$" is adapted from the Farey sequence denoting $a/b \oplus c/d = (a+c)/(b+d)$.

Following the same procedure in proving Theorem 1, it is easy to derive the limiting distribution of the first term on the right-hand side of (47) to be

$$\frac{\frac{\boldsymbol{\upsilon}_i'\mathbf{M}_p\boldsymbol{s}_{i,-1}}{T}}{\left(\frac{\boldsymbol{\upsilon}_i'\overline{\mathbf{M}}_{i,p}\boldsymbol{\upsilon}_i}{T-4}\right)^{1/2}\left(\frac{\boldsymbol{s}_{i,-1}'\mathbf{M}_p\boldsymbol{s}_{i,-1}}{T^2}\right)^{1/2}} \xrightarrow{N} \frac{\frac{\boldsymbol{\varepsilon}_i'\mathbf{s}_{i,-1}}{\sigma_i^2T} - (\mathbf{q}_{iT}'\boldsymbol{\Gamma}_T'^P)(\boldsymbol{\Gamma}_T^P\boldsymbol{\Psi}_{fT}\boldsymbol{\Gamma}_T'^P)^{-1}(\boldsymbol{\Gamma}_T^P\mathbf{h}_{iT})}{J_1^P\times J_2^P},$$

where J_1^P, J_2^P and $\boldsymbol{\Gamma}_T^P$ are defined in (19)-(21) and $\mathbf{q}_{iT}$, $\boldsymbol{\Psi}_{fT}$, $\mathbf{h}_{iT}$ are defined in (12) and (14).

We next consider the limiting behavior of the bias terms $\aleph_{i,T}$ as $N\to\infty$. Using Lemmas 1 and 2 as well as the results of proving Theorem 1 in Pesaran (2007) and this paper, the first element of $\aleph_{i,T}$ is $O_p(T^{-1/2})$. This is because it can be factored by the scaling matrix $\mathbf{H}=diag[T^{-1/2},\ T^{-1/2},\ T^{-1}]$ as follows:

$$\frac{\boldsymbol{\Upsilon}'^*\mathbf{M}_p\boldsymbol{s}_{i,-1}}{\omega_iT} = \frac{\boldsymbol{\Upsilon}'^*\boldsymbol{s}_{i,-1}}{\omega_iT} - \left(\frac{\boldsymbol{\Upsilon}'^*\mathbf{PH}}{\omega_i}\right)(\mathbf{HP}'\mathbf{PH})^{-1}\left(\frac{\mathbf{HP}'\boldsymbol{s}_{i,-1}}{T}\right),$$

the order of the magnitude of each term on the right-hand side of the above equation is $\frac{\boldsymbol{\Upsilon}'^*\boldsymbol{s}_{i,-1}}{\omega_iT}\equiv O_p(T^{-1/2})$, $\frac{\boldsymbol{\Upsilon}'^*\mathbf{PH}}{\omega_i} = \left[\frac{\Delta\overline{\mathbf{y}}'\boldsymbol{\Upsilon}^*}{\omega_i\sqrt{T}}\ \frac{\boldsymbol{\tau}'\boldsymbol{\Upsilon}^*}{\omega_i\sqrt{T}}\ \frac{\overline{\mathbf{y}}_{-1}'\boldsymbol{\Upsilon}^*}{\omega_iT}\right]\equiv\left[O_p(T^{-1})\ O_p(T^{-1/2})\ O_p(T^{-1/2})\right]$, $\frac{\mathbf{HP}'\boldsymbol{s}_{i,-1}}{T}$ and $\mathbf{HP}'\mathbf{PH}$ are $O_p(1)$. Hence, $\frac{\boldsymbol{\Upsilon}'^*\mathbf{M}_p\boldsymbol{s}_{i,-1}}{\omega_iT}$ is $O_p(T^{-1/2})$. Likewise, it is easy to show that the second and third elements of $\aleph_{i,T}$ are $O_p(T^{-1/2})$ and $O_p(T^{-1})$, respectively. Therefore, $\aleph_{i,T}$ is $O_p(T^{-1/2})$. Similarly, we can show that $\hbar_{i,T}$ is $O_p(T^{-1/4})$ and then conclude, using (47), that

$$t_i^{P,B}(N,T)\xrightarrow{N}\frac{\frac{\boldsymbol{\varepsilon}_i'\mathbf{s}_{i,-1}}{\sigma_i^2T} - (\mathbf{q}_{iT}'\boldsymbol{\Gamma}_T'^P)(\boldsymbol{\Gamma}_T^P\boldsymbol{\Psi}_{fT}\boldsymbol{\Gamma}_T'^P)^{-1}(\boldsymbol{\Gamma}_T^P\mathbf{h}_{iT})}{J_1^P\times J_2^P+O(T^{-1/4})}+\frac{O(T^{-1/2})}{J_1^P\times J_2^P+O(T^{-1/4})}.$$

If next to N, T also tends to infinity then, using the convergence results in proving Theorem 2, we have

$$t_i^{P,B}(N,T)\xrightarrow{(N,T)_{seq}}\frac{\int_0^1W_i(r)dW_i(r)-\mathbf{q}_{if}^{P'}\left(\boldsymbol{\Psi}_f^P\right)^{-1}\mathbf{h}_{if}^P}{\left(\int_0^1W_i^2(r)d(r)-\mathbf{h}_{if}^{P'}\left(\boldsymbol{\Psi}_f^P\right)^{-1}\mathbf{h}_{if}^P\right)^{1/2}}.$$

This completes the proof of this corollary.

Proof of Theorem 3:

Following the procedures of proving Theorems 1, 2 and the Theorem 5.1 in Pesaran (2007), it is simple to prove Theorem 3. The detailed proof is available from the authors upon request.

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Table 1. Simulated M_1 and M_2

	k =1		k =2		k =3		k =4		k =5	
	M_1	M_2	M_1	M_2	M_1	M_2	M_1	M_2	M_1	M_2
C1	7.728	4.563	6.831	4.551	6.444	4.332	6.307	4.259	6.208	4.222
C2	7.163	1.999	7.060	2.942	6.667	2.831	6.450	2.716	6.352	2.700
C3	6.925	0.473	7.441	1.864	7.084	1.937	6.819	1.860	6.679	1.823

Notes: Three different cases are considered and they are the model without an intercept and trend (C1), with an intercept (C2) and with an intercept and trend (C3), respectively. Besides, the deterministic component in all cases includes a Fourier function to capture smoothing breaks. M_1 and M_2 for different cases are simulated based on N=300 and T=500 with 50,000 replications. The individual series were generated as $y_{it} = y_{it-1} + f_t + \varepsilon_{it}$ for $i=1,\ldots,N$ and $t=1, \ldots,T$ with y_{i0}, f_t and ε_{it} as i.i.d.N(0,1).

Table 2(a). Critical values of average of individual breaks and cross-sectional dependence augmented Dickey-Fuller distribution

(Case I: no intercept and no trend)

	k=1						*k*=2						*k*=3						*k*=4						*k*=5					
T/N	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200
1%																														
30	-2.83	-2.64	-2.59	-2.52	-2.48	-2.45	-2.19	-1.99	-1.92	-1.84	-1.78	-1.75	-2.00	-1.82	-1.74	-1.67	-1.63	-1.60	-1.93	-1.76 (-1.75)	-1.67	-1.62	-1.56	-1.53	-1.89	-1.71	-1.65	-1.59	-1.54	-1.51
50	-2.81	-2.63	-2.59	-2.52	-2.48	-2.45	-2.21	-2.02	-1.94	-1.88	-1.83	-1.79	-2.04	-1.86	-1.79	-1.73	-1.67	-1.65	-1.98	-1.80	-1.73	-1.67	-1.61	-1.59	-1.94	-1.76	-1.70	-1.64	-1.60	-1.57
70	-2.80	-2.65	-2.58	-2.53	-2.48	-2.46	-2.21	-2.03	-1.96	-1.90	-1.84	-1.81	-2.06	-1.88	-1.81	-1.75	-1.70	-1.67	-1.98	-1.82	-1.75	-1.70	-1.64	-1.62	-1.95	-1.79	-1.73	-1.66	-1.62	-1.59
100	-2.81	-2.64	-2.58	-2.53	-2.49	-2.46	-2.23	-2.04	-1.98	-1.91	-1.85	-1.82	-2.06	-1.89	-1.82	-1.76	-1.71	-1.68	-2.00	-1.82	-1.77	-1.71	-1.66	-1.63	-1.96	-1.81	-1.74	-1.68	-1.63	-1.61
200	-2.80	-2.64	-2.59	-2.52	-2.49	-2.46	-2.24	-2.06	-2.00	-1.92	-1.87	-1.84	-2.09	-1.91	-1.84	-1.78	-1.73	-1.70	-2.01	-1.85	-1.79	-1.73	-1.68	-1.65	-1.99	-1.82	-1.76	-1.71	-1.66	-1.63
400	-2.79	-2.64	-2.57	-2.53	-2.49	-2.47	-2.24	-2.07	-2.00	-1.93	-1.88	-1.85	-2.09	-1.91	-1.85	-1.79	-1.74	-1.71	-2.03	-1.86	-1.79	-1.73	-1.69	-1.66	-1.98	-1.82	-1.77	-1.71	-1.67	-1.64
5%																														
30	-2.55	-2.43	-2.39	-2.36	-2.33	-2.32	-1.90	-1.77	-1.72	-1.69	-1.65	-1.64	-1.72	-1.61	-1.56	-1.53	-1.50	-1.49	-1.66	-1.56	-1.51	-1.47	-1.44	-1.43	-1.63	-1.52	-1.48	-1.45	-1.42	-1.41
50	-2.54	-2.43	-2.40	-2.37	-2.35	-2.33	-1.93	-1.80	-1.76	-1.72	-1.70	-1.68	-1.77	-1.66	-1.62	-1.58	-1.56	-1.54	-1.72	-1.60	-1.57	-1.53	-1.50	-1.49	-1.69	-1.57	-1.53	-1.50	-1.48	-1.47
70	-2.54	-2.44	-2.40	-2.37	-2.35	-2.34	-1.93	-1.83	-1.78	-1.75	-1.71	-1.70	-1.79	-1.68	-1.64	-1.60	-1.58	-1.56	-1.72	-1.63	-1.58	-1.55	-1.53	-1.52	-1.70	-1.60	-1.56	-1.53	-1.50	-1.49
100	-2.54	-2.44	-2.40	-2.37	-2.35	-2.34	-1.95	-1.83	-1.80	-1.76	-1.73	-1.72	-1.80	-1.69	-1.65	-1.62	-1.59	-1.58	-1.74	-1.64	-1.60	-1.57	-1.54	-1.53	-1.72	-1.62	-1.58	-1.55	-1.52	-1.51
200	-2.54	-2.44	-2.40	-2.38	-2.36	-2.35	-1.96	-1.85	-1.81	-1.77	-1.74	-1.74	-1.82	-1.72	-1.67	-1.63	-1.61	-1.60	-1.76	-1.66	-1.62	-1.59	-1.56	-1.55	-1.74	-1.63	-1.60	-1.57	-1.54	-1.53
400	-2.54	-2.45	-2.40	-2.38	-2.36	-2.35	-1.97	-1.85	-1.82	-1.78	-1.75	-1.74	-1.83	-1.72	-1.68	-1.64	-1.62	-1.61	-1.77	-1.67	-1.63	-1.60	-1.58	-1.56	-1.75	-1.64	-1.61	-1.57	-1.55	-1.54
10%																														
30	-2.38	-2.30	-2.27	-2.25	-2.24	-2.23	-1.73	-1.64	-1.61	-1.59	-1.57	-1.56	-1.57	-1.49	-1.46	-1.44	-1.42	-1.41	-1.51	-1.44	-1.40	-1.38	-1.37	-1.36	-1.49	-1.41	-1.37	-1.36	-1.34	-1.33
50	-2.38	-2.30	-2.28	-2.26	-2.25	-2.25	-1.77	-1.68	-1.65	-1.63	-1.61	-1.61	-1.62	-1.54	-1.51	-1.49	-1.47	-1.47	-1.57	-1.49	-1.46	-1.44	-1.42	-1.42	-1.55	-1.46	-1.43	-1.41	-1.40	-1.40
70	-2.38	-2.31	-2.28	-2.27	-2.25	-2.25	-1.78	-1.70	-1.67	-1.65	-1.63	-1.62	-1.64	-1.56	-1.53	-1.51	-1.50	-1.49	-1.58	-1.51	-1.48	-1.46	-1.45	-1.44	-1.55	-1.49	-1.46	-1.44	-1.42	-1.42
100	-2.38	-2.31	-2.28	-2.27	-2.26	-2.25	-1.80	-1.71	-1.68	-1.66	-1.65	-1.64	-1.65	-1.58	-1.55	-1.53	-1.51	-1.51	-1.60	-1.52	-1.50	-1.48	-1.47	-1.46	-1.58	-1.50	-1.48	-1.46	-1.44	-1.43
200	-2.38	-2.31	-2.29	-2.27	-2.26	-2.25	-1.81	-1.73	-1.70	-1.68	-1.66	-1.66	-1.67	-1.60	-1.57	-1.54	-1.53	-1.53	-1.62	-1.54	-1.52	-1.50	-1.48	-1.48	-1.60	-1.52	-1.50	-1.48	-1.46	-1.45
400	-2.38	-2.32	-2.29	-2.27	-2.26	-2.26	-1.81	-1.73	-1.71	-1.68	-1.67	-1.66	-1.68	-1.60	-1.58	-1.55	-1.55	-1.53	-1.63	-1.56	-1.53	-1.51	-1.50	-1.49	-1.61	-1.53	-1.51	-1.49	-1.47	-1.47

Notes: Numbers in the table are critical values of the *BCIPS* and *BCIPS** statistics. The critical values of the *BCIPS** statistic are reported in parentheses when they are different from those of the *BCIPS* statistic. Critical values are simulated with 50,000 replications based on the OLS regression of Δy_{it} on $\Delta \sin(2\pi kt/T)$, $\Delta \cos(2\pi kt/T)$, y_{it-1}, $\bar{y}_{t-1}$ and $\Delta\bar{y}_{t-1}$, over the sample *t=1, …, T*, where $\bar{y}_t = N^{-1}\sum_{j=1}^{N} y_{jt}$. The individual series were generated as $y_{it} = y_{it-1} + f_t + \varepsilon_{it}$ for *i=1,…, N* and *t=1,…, T* with y_{i0}, f_t and ε_{it} as i.i.d.*N*(0,1). $BCIPS^*(N,T) = N^{-1}\sum_{i=1}^{N} t_i^*(N,T)$, where $t_i^*(N,T)$ is the truncated t-statistic described in (23).

Table 2(b). Critical values of average of individual breaks and cross-sectional dependence augmented Dickey-Fuller distribution (Case II: intercept only)

	k=1						k=2						k=3						k=4						k=5					
T/N	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200
1%																														
30	-3.45	-3.24	-3.16	-3.10 (-3.07)	-3.03	-3.01	-2.98	-2.74 (-2.73)	-2.66	-2.59 (-2.58)	-2.51	-2.47	-2.69	-2.47	-2.38	-2.31	-2.24	-2.20	-2.56	-2.35	-2.27	-2.20	-2.13	-2.10	-2.50	-2.30	-2.22	-2.15	-2.09	-2.05
50	-3.41	-3.21	-3.15	-3.08	-3.03	-3.00	-2.95	-2.75	-2.65	-2.58	-2.51	-2.48	-2.71	-2.49	-2.42	-2.35	-2.28	-2.25	-2.60	-2.40	-2.33	-2.26	-2.19	-2.16	-2.54	-2.35	-2.28	-2.22	-2.15	-2.12
70	-3.37	-3.20	-3.13	-3.08	-3.03	-3.00	-2.94	-2.74	-2.66	-2.59	-2.53	-2.49	-2.71	-2.51	-2.43	-2.37	-2.31	-2.27	-2.61	-2.43	-2.35	-2.28	-2.22	-2.19	-2.57	-2.39	-2.31	-2.24	-2.18	-2.15
100	-3.37	-3.19	-3.13	-3.08	-3.03	-3.00	-2.95	-2.74	-2.67	-2.60	-2.53	-2.50	-2.72	-2.52	-2.45	-2.38	-2.32	-2.29	-2.63	-2.43	-2.36	-2.30	-2.24	-2.21	-2.58	-2.39	-2.32	-2.26	-2.20	-2.17
200	-3.35	-3.18	-3.12	-3.07	-3.02	-3.00	-2.94	-2.74	-2.67	-2.60	-2.54	-2.50	-2.72	-2.54	-2.46	-2.40	-2.34	-2.30	-2.64	-2.45	-2.38	-2.31	-2.26	-2.23	-2.59	-2.41	-2.34	-2.28	-2.22	-2.19
400	-3.34	-3.18	-3.11	-3.07	-3.02	-3.00	-2.93	-2.75	-2.67	-2.60	-2.54	-2.51	-2.74	-2.53	-2.47	-2.40	-2.34	-2.31	-2.65	-2.47	-2.38	-2.33	-2.27	-2.24	-2.60	-2.42	-2.36	-2.29	-2.23	-2.20
5%																														
30	-3.19	-3.05	-2.99	-2.96	-2.92	-2.90	-2.68	-2.53	-2.47	-2.43	-2.38	-2.36	-2.42	-2.27	-2.21	-2.17	-2.12	-2.10	-2.31	-2.17	-2.12	-2.07	-2.03	-2.01	-2.26	-2.12	-2.07	-2.02	-1.99	-1.96
50	-3.16	-3.03	-2.99	-2.95	-2.92	-2.91	-2.68	-2.54	-2.49	-2.44	-2.40	-2.38	-2.45	-2.32	-2.26	-2.22	-2.18	-2.16	-2.37	-2.23	-2.18	-2.13	-2.10	-2.08	-2.32	-2.18	-2.13	-2.09	-2.06	-2.04
70	-3.15	-3.03	-2.99	-2.95	-2.92	-2.91	-2.69	-2.55	-2.50	-2.45	-2.42	-2.39	-2.47	-2.33	-2.28	-2.24	-2.20	-2.18	-2.38	-2.25	-2.20	-2.16	-2.13	-2.11	-2.34	-2.21	-2.16	-2.12	-2.08	-2.07
100	-3.15	-3.02	-2.98	-2.95	-2.92	-2.91	-2.70	-2.55	-2.50	-2.46	-2.42	-2.40	-2.48	-2.35	-2.30	-2.25	-2.21	-2.20	-2.39	-2.27	-2.22	-2.17	-2.14	-2.13	-2.36	-2.23	-2.18	-2.14	-2.10	-2.09
200	-3.14	-3.02	-2.98	-2.95	-2.92	-2.91	-2.69	-2.56	-2.51	-2.46	-2.43	-2.41	-2.50	-2.37	-2.32	-2.27	-2.24	-2.22	-2.42	-2.28	-2.24	-2.20	-2.16	-2.15	-2.38	-2.25	-2.20	-2.16	-2.13	-2.11
400	-3.14	-3.02	-2.98	-2.94	-2.92	-2.91	-2.69	-2.56	-2.51	-2.46	-2.43	-2.41	-2.49	-2.36	-2.32	-2.27	-2.24	-2.23	-2.42	-2.30	-2.25	-2.21	-2.18	-2.16	-2.38	-2.26	-2.21	-2.17	-2.14	-2.12
10%																														
30	-3.05	-2.95	-2.91	-2.88	-2.85	-2.84	-2.53	-2.41	-2.37	-2.34	-2.31	-2.29	-2.28	-2.17	-2.12	-2.09	-2.06	-2.05	-2.18	-2.07	-2.03	-2.00	-1.97	-1.96	-2.13	-2.03 (-2.02)	-1.98	-1.95	-1.93	-1.91
50	-3.03	-2.93	-2.90	-2.87	-2.85	-2.85	-2.54	-2.43	-2.39	-2.36	-2.33	-2.32	-2.32	-2.21	-2.18	-2.14	-2.12	-2.10	-2.24	-2.13	-2.09	-2.06	-2.04	-2.02	-2.19	-2.09	-2.05	-2.02	-2.00	-1.99
70	-3.03	-2.94	-2.90	-2.88	-2.85	-2.84	-2.55	-2.44	-2.40	-2.37	-2.34	-2.33	-2.34	-2.23	-2.19	-2.16	-2.14	-2.13	-2.26	-2.16	-2.12	-2.09	-2.06	-2.05	-2.22	-2.12	-2.08	-2.05	-2.03	-2.01
100	-3.03	-2.93	-2.90	-2.87	-2.86	-2.84	-2.55	-2.44	-2.40	-2.37	-2.35	-2.34	-2.35	-2.25	-2.21	-2.18	-2.15	-2.14	-2.27	-2.17	-2.14	-2.10	-2.08	-2.07	-2.23	-2.14	-2.10	-2.07	-2.05	-2.04
200	-3.02	-2.93	-2.90	-2.87	-2.86	-2.84	-2.56	-2.45	-2.42	-2.38	-2.36	-2.35	-2.37	-2.27	-2.23	-2.20	-2.17	-2.16	-2.29	-2.19	-2.16	-2.13	-2.10	-2.09	-2.26	-2.16	-2.12	-2.09	-2.07	-2.06
400	-3.02	-2.93	-2.90	-2.87	-2.85	-2.85	-2.56	-2.45	-2.42	-2.39	-2.36	-2.35	-2.37	-2.27	-2.23	-2.20	-2.18	-2.17	-2.30	-2.20	-2.17	-2.14	-2.11	-2.10	-2.27	-2.17	-2.13	-2.10	-2.08	-2.07

Notes: Numbers in the table are critical values of the *BCIPS* and *BCIPS** statistics. The critical values of *the BCIPS** are reported in parentheses when they are different from those of the *BCIPS* statistic. Critical values are simulated with 50,000 replications based on the OLS regression of Δy_{it} on c, $\Delta\sin(2\pi kt/T)$, $\Delta\cos(2\pi kt/T)$, y_{it-1}, $\bar{y}_{t-1}$ and $\Delta\bar{y}_{t-1}$, over the sample $t=1,\ldots,T$, where $\bar{y}_t = N^{-1}\sum_{j=1}^{N} y_{jt}$. The individual series were generated as $y_{it} = y_{it-1} + f_t + \varepsilon_{it}$ for $i=1,\ldots,N$ and $t=1,\ldots,T$ with y_{i0}, f_t and ε_{it} as i.i.d.$N(0,1)$. $BCIPS^*(N,T) = N^{-1}\sum_{i=1}^{N} t_i^*(N,T)$, where $t_i^*(N,T)$ is the truncated t-statistic described in (23).

Table 2(c). Critical values of average of individual breaks and cross-sectional dependence augmented Dickey-Fuller distribution (Case III: intercept and trend)

	k=1						*k* =2						*k* =3						*k* =4						*k* =5					
T/N	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200
1%																														
30	-3.96	-3.75	-3.67	-3.58	-3.52	-3.47	-3.69	-3.48	-3.39	-3.31	-3.25	-3.22	-3.33	-3.10	-3.01	-2.93	-2.85	-2.81	-3.14	-2.93	-2.83	-2.75	-2.68	-2.64	-3.05	-2.83	-2.74	-2.66	-2.60	-2.55
					(-3.51)																									
50	-3.90	-3.70	-3.62	-3.54	-3.48	-3.44	-3.62	-3.43	-3.35	-3.29	-3.23	-3.20	-3.33	-3.11	-3.02	-2.95	-2.87	-2.84	-3.17	-2.96	-2.88	-2.80	-2.73	-2.70	-3.08	-2.89	-2.80	-2.73	-2.66	-2.63
70	-3.87	-3.68	-3.60	-3.53	-3.47	-3.43	-3.62	-3.41	-3.33	-3.28	-3.23	-3.19	-3.32	-3.11	-3.03	-2.96	-2.89	-2.85	-3.18	-2.98	-2.90	-2.82	-2.76	-2.72	-3.11	-2.90	-2.83	-2.76	-2.69	-2.65
100	-3.85	-3.67	-3.59	-3.53	-3.46	-3.42	-3.59	-3.40	-3.34	-3.27	-3.22	-3.19	-3.32	-3.12	-3.04	-2.96	-2.90	-2.86	-3.18	-2.99	-2.91	-2.83	-2.77	-2.74	-3.12	-2.92	-2.84	-2.78	-2.72	-2.67
200	-3.84	-3.66	-3.59	-3.52	-3.45	-3.41	-3.57	-3.40	-3.32	-3.27	-3.21	-3.19	-3.32	-3.12	-3.05	-2.97	-2.91	-2.87	-3.20	-3.01	-2.93	-2.85	-2.79	-2.76	-3.13	-2.94	-2.87	-2.79	-2.73	-2.70
400	-3.84	-3.65	-3.58	-3.51	-3.45	-3.41	-3.57	-3.39	-3.32	-3.26	-3.21	-3.18	-3.32	-3.13	-3.04	-2.97	-2.91	-2.88	-3.20	-3.01	-2.93	-2.86	-2.80	-2.76	-3.13	-2.94	-2.87	-2.80	-2.74	-2.71
5%																														
30	-3.73	-3.58	-3.52	-3.46	-3.42	-3.39	-3.40	-3.26	-3.21	-3.16	-3.12	-3.10	-3.05	-2.90	-2.84	-2.78	-2.73	-2.71	-2.89	-2.73	-2.67	-2.62	-2.57	-2.54	-2.79	-2.65	-2.59	-2.53	-2.49	-2.46
50	-3.69	-3.55	-3.49	-3.44	-3.40	-3.37	-3.37	-3.25	-3.19	-3.15	-3.11	-3.09	-3.08	-2.93	-2.87	-2.82	-2.77	-2.75	-2.94	-2.79	-2.73	-2.68	-2.63	-2.61	-2.86	-2.72	-2.66	-2.61	-2.57	-2.55
70	-3.68	-3.54	-3.49	-3.44	-3.39	-3.36	-3.37	-3.23	-3.18	-3.14	-3.11	-3.09	-3.08	-2.94	-2.88	-2.84	-2.79	-2.77	-2.96	-2.81	-2.76	-2.71	-2.66	-2.64	-2.89	-2.75	-2.69	-2.65	-2.60	-2.58
100	-3.67	-3.53	-3.48	-3.44	-3.39	-3.36	-3.36	-3.23	-3.18	-3.14	-3.10	-3.09	-3.09	-2.95	-2.90	-2.84	-2.80	-2.78	-2.97	-2.83	-2.77	-2.72	-2.68	-2.66	-2.90	-2.77	-2.71	-2.67	-2.62	-2.60
200	-3.66	-3.53	-3.48	-3.43	-3.39	-3.36	-3.35	-3.22	-3.18	-3.14	-3.11	-3.09	-3.10	-2.96	-2.90	-2.86	-2.82	-2.79	-2.98	-2.84	-2.79	-2.74	-2.70	-2.68	-2.92	-2.79	-2.74	-2.68	-2.65	-2.62
400	-3.65	-3.53	-3.48	-3.43	-3.38	-3.36	-3.35	-3.22	-3.18	-3.13	-3.10	-3.08	-3.10	-2.96	-2.91	-2.86	-2.82	-2.80	-2.99	-2.85	-2.80	-2.75	-2.71	-2.69	-2.93	-2.79	-2.75	-2.70	-2.66	-2.63
10%																														
30	-3.60	-3.49	-3.45	-3.40	-3.36	-3.34	-3.26	-3.14	-3.11	-3.07	-3.05	-3.03	-2.91	-2.79	-2.75	-2.70	-2.67	-2.65	-2.75	-2.63	-2.59	-2.55	-2.51	-2.49	-2.66	-2.55	-2.51	-2.47	-2.43	-2.41
			(-3.44)						(-3.10)																					
50	-3.58	-3.47	-3.43	-3.39	-3.35	-3.34	-3.24	-3.14	-3.10	-3.06	-3.04	-3.03	-2.95	-2.83	-2.79	-2.75	-2.71	-2.70	-2.81	-2.70	-2.65	-2.62	-2.58	-2.56	-2.74	-2.63	-2.59	-2.55	-2.52	-2.50
70	-3.57	-3.47	-3.43	-3.39	-3.35	-3.33	-3.24	-3.13	-3.09	-3.06	-3.04	-3.02	-2.96	-2.85	-2.80	-2.77	-2.73	-2.71	-2.84	-2.72	-2.68	-2.65	-2.61	-2.59	-2.77	-2.66	-2.62	-2.58	-2.55	-2.53
100	-3.57	-3.47	-3.42	-3.39	-3.35	-3.33	-3.23	-3.13	-3.10	-3.06	-3.04	-3.03	-2.97	-2.86	-2.82	-2.78	-2.74	-2.73	-2.85	-2.74	-2.70	-2.66	-2.63	-2.61	-2.79	-2.68	-2.64	-2.61	-2.57	-2.55
	(-3.56)																													
200	-3.56	-3.46	-3.42	-3.38	-3.35	-3.33	-3.23	-3.13	-3.09	-3.06	-3.04	-3.03	-2.98	-2.87	-2.83	-2.79	-2.76	-2.74	-2.87	-2.76	-2.72	-2.68	-2.65	-2.63	-2.81	-2.70	-2.67	-2.63	-2.60	-2.58
400	-3.56	-3.46	-3.42	-3.38	-3.35	-3.33	-3.23	-3.13	-3.09	-3.06	-3.04	-3.03	-2.98	-2.88	-2.84	-2.79	-2.77	-2.75	-2.88	-2.77	-2.73	-2.69	-2.66	-2.64	-2.82	-2.72	-2.68	-2.64	-2.61	-2.59

Notes: Numbers in the table are critical values of the *BCIPS* and *BCIPS** statistics. The critical values of *the BCIPS** statistic are reported in parentheses when they are different from those of the *BCIPS* statistic. Critical values are simulated with 50,000 replications based on the OLS regression of Δy_{it} on *c*, *t*, $\Delta\sin(2\pi kt/T)$, $\Delta\cos(2\pi kt/T)$, y_{it-1}, $\bar{y}_{t-1}$ and $\Delta\bar{y}_{t-1}$, over the sample *t=1,…,T*, where $\bar{y}_t = N^{-1}\sum_{j=1}^{N} y_{jt}$. The individual series were generated as $y_{it} = y_{it-1} + f_t + \varepsilon_{it}$ for *i=1,…,N* and *t=1,…,T* with y_{i0}, f_t and ε_{it} as i.i.d.*N*(0,1). $BCIPS^*(N,T) = N^{-1}\sum_{i=1}^{N} t_i^*(N,T)$, where $t_i^*(N,T)$ is the truncated t-statistic described in (23).

Table 3. Size and power of the panel unit-root test allowing for smoothing breaks: no serial correlation, high cross-sectional dependence, breaks with high amplitude (Intercept case)

	k =1						k =2						k =3					
T/N	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200
									Size									
15	0.071	0.078	0.064	0.071	0.068	0.062	0.066	0.091	0.083	0.093	0.082	0.091	0.062	0.068	0.059	0.060	0.069	0.056
30	0.061	0.055	0.059	0.055	0.051	0.057	0.066	0.061	0.070	0.058	0.071	0.062	0.061	0.048	0.064	0.058	0.056	0.059
50	0.048	0.059	0.051	0.053	0.049	0.049	0.058	0.064	0.053	0.060	0.056	0.057	0.049	0.063	0.051	0.052	0.054	0.062
100	0.056	0.066	0.054	0.049	0.054	0.048	0.058	0.059	0.052	0.054	0.060	0.053	0.064	0.059	0.055	0.048	0.056	0.055
200	0.055	0.057	0.049	0.054	0.047	0.052	0.055	0.055	0.043	0.051	0.053	0.052	0.058	0.047	0.048	0.052	0.051	0.052
400	0.047	0.051	0.057	0.057	0.050	0.054	0.055	0.050	0.051	0.053	0.052	0.047	0.047	0.054	0.053	0.049	0.051	0.044
									Power									
15	0.094	0.108	0.110	0.115	0.130	0.116	0.098	0.112	0.109	0.117	0.133	0.123	0.081	0.104	0.095	0.099	0.113	0.108
30	0.140	0.171	0.192	0.192	0.206	0.228	0.157	0.192	0.216	0.246	0.279	0.335	0.174	0.212	0.248	0.274	0.304	0.366
50	0.272	0.330	0.410	0.537	0.603	0.735	0.393	0.438	0.593	0.728	0.792	0.914	0.422	0.489	0.677	0.826	0.889	0.984
100	0.671	0.931	0.982	0.999	1.000	1.000	0.869	0.994	0.999	1.000	1.000	1.000	0.922	0.999	1.000	1.000	1.000	1.000
200	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: Numbers in the table denote the sizes of the $BCIPS^{*}$ *test*. Data are generated by: $y_{it} = (1-\phi_i L)(\mu_i + \alpha_{i,1}\sin(2\pi kt/T) + \alpha_{i,2}\cos(2\pi kt/T)) + \phi_i y_{it-1} + \gamma_i f_t + \varepsilon_{it}$, for *i=1,…,N and t=1,…,T,* where y_{i0}, μ_i and $f_t \sim$ i.i.d.$N(0.1)$; $\gamma_i \sim$ i.i.d.$U[-1,3]$ (high cross-sectional dependence); $\alpha_{i,1}, \alpha_{i,2} \sim$ i.i.d.$U[1,2]$ (high amplitude); $\varepsilon_{it} \sim$ i.i.d.$N(0,\sigma_i^2)$ with $\sigma_i^2 \sim$ i.i.d$U[0.5,1.5]$. Size (under the null $\phi_i = 1$) and power (under the null $\phi_i \sim$ i.i.d.$U[0.85, 0.95]$) are computed at the 5% nominal level based on the following regression : $\Delta y_{it} = c_{i,0} + c_{i,1}\Delta\sin(2k\pi t/T) + c_{i,2}\Delta\cos(2k\pi t/T) + c_{i,3}\bar{y}_{t-1} + c_{i,4}\Delta\bar{y}_t + b_i y_{it-1} + e_{it}$, *i=1,…,N, t=1,…,T,* where $\bar{y}_t = N^{-1}\sum_{j=1}^{N} y_{jt}$, $\Delta\bar{y}_t = \bar{y}_t - \bar{y}_{t-1}$. $BCIPS^{*}(N,T) = N^{-1}\sum_{i=1}^{N} t_i^{*}(N,T)$, where $t_i^{*}(N,T)$ is the truncated t-statistic described in (23).

Table 4. Size and power of the panel unit-root test: high cross-sectional dependence, breaks with high amplitude, serially correlated errors (Intercept case)

	k =1						k =2						k =3					
T/N	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200
							Size: Positive residual serial correlations											
30	0.048	0.047	0.045	0.037	0.034	0.038	0.035	0.021	0.018	0.012	0.007	0.006	0.021	0.008	0.007	0.004	0.001	0.000
50	0.048	0.058	0.041	0.047	0.042	0.044	0.038	0.037	0.024	0.022	0.018	0.013	0.025	0.020	0.017	0.012	0.004	0.003
100	0.051	0.065	0.053	0.046	0.048	0.043	0.044	0.045	0.038	0.034	0.033	0.025	0.043	0.040	0.028	0.024	0.019	0.015
200	0.057	0.053	0.045	0.051	0.047	0.049	0.053	0.044	0.038	0.038	0.042	0.038	0.048	0.037	0.036	0.035	0.034	0.028
400	0.043	0.048	0.056	0.059	0.051	0.051	0.051	0.046	0.048	0.046	0.045	0.040	0.043	0.048	0.047	0.043	0.038	0.034
							Size: Negative residual serial correlations											
30	0.053	0.065	0.060	0.057	0.045	0.053	0.048	0.055	0.043	0.043	0.050	0.050	0.037	0.033	0.026	0.023	0.023	0.019
50	0.054	0.053	0.051	0.051	0.054	0.053	0.053	0.050	0.049	0.048	0.052	0.049	0.045	0.039	0.038	0.029	0.026	0.027
100	0.063	0.070	0.052	0.052	0.054	0.053	0.052	0.067	0.047	0.051	0.041	0.053	0.058	0.061	0.046	0.044	0.040	0.039
200	0.068	0.052	0.055	0.045	0.054	0.057	0.058	0.056	0.049	0.049	0.052	0.047	0.064	0.039	0.049	0.043	0.043	0.043
400	0.052	0.046	0.053	0.053	0.048	0.053	0.053	0.053	0.050	0.056	0.054	0.048	0.053	0.046	0.046	0.048	0.049	0.051
							Power: Positive residual serial correlations											
30	0.126	0.135	0.157	0.140	0.154	0.161	0.089	0.075	0.088	0.068	0.078	0.080	0.054	0.036	0.037	0.025	0.023	0.016
50	0.204	0.293	0.348	0.353	0.422	0.437	0.196	0.268	0.334	0.321	0.404	0.428	0.155	0.219	0.252	0.246	0.308	0.352
100	0.590	0.774	0.941	0.955	0.999	1.000	0.789	0.915	0.993	0.997	1.000	1.000	0.812	0.939	0.997	0.999	1.000	1.000
200	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
							Power: Negative residual serial correlations											
30	0.108	0.148	0.144	0.139	0.132	0.116	0.106	0.166	0.168	0.181	0.178	0.166	0.092	0.162	0.137	0.145	0.140	0.131
50	0.260	0.298	0.348	0.346	0.482	0.464	0.374	0.416	0.501	0.534	0.722	0.700	0.414	0.444	0.541	0.590	0.817	0.829
100	0.390	0.893	0.903	0.994	1.000	1.000	0.572	0.987	0.993	1.000	1.000	1.000	0.656	0.996	0.997	1.000	1.000	1.000
200	0.990	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: Numbers in the table denote the sizes of the *BCIPS** *statistic*. Data are generated by: $y_{it} = (1-\phi_i L)(\mu_i + \alpha_{i,1}\sin(2\pi kt/T) + \alpha_{i,2}\cos(2\pi kt/T)) + \phi_i y_{it-1} + \gamma_i f_t + \varepsilon_{it}$ and $\varepsilon_{it} = \rho_i \varepsilon_{it-1} + e_{it}$, for *i=1,…,N* and *t=1,…,T. See Notes on Table 3 for parameters and initial values setting.*

Table 5. Size and power of the panel unit-root test: no serial correlation, high cross-sectional dependence, breaks with high amplitude (Trend case)

	k =1						*k* =2						*k* =3					
T/N	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200
							Size											
30	0.059	0.051	0.052	0.048	0.047	0.045	0.070	0.066	0.072	0.076	0.074	0.082	0.068	0.057	0.070	0.064	0.063	0.067
50	0.047	0.055	0.054	0.052	0.053	0.048	0.060	0.064	0.058	0.061	0.064	0.069	0.054	0.062	0.056	0.064	0.062	0.057
100	0.057	0.062	0.053	0.053	0.050	0.048	0.065	0.060	0.057	0.056	0.058	0.056	0.063	0.066	0.052	0.054	0.058	0.060
200	0.058	0.047	0.049	0.048	0.054	0.047	0.058	0.054	0.049	0.049	0.053	0.048	0.059	0.054	0.053	0.048	0.051	0.051
400	0.051	0.055	0.056	0.052	0.051	0.046	0.055	0.055	0.052	0.052	0.056	0.049	0.045	0.055	0.049	0.056	0.051	0.054
							Power											
30	0.074	0.069	0.069	0.063	0.065	0.072	0.096	0.107	0.103	0.102	0.107	0.132	0.108	0.126	0.127	0.128	0.136	0.176
50	0.090	0.136	0.120	0.150	0.178	0.182	0.142	0.227	0.189	0.275	0.253	0.270	0.173	0.325	0.269	0.443	0.456	0.552
100	0.315	0.572	0.763	0.884	0.973	0.997	0.558	0.838	0.955	0.989	0.999	1.000	0.656	0.933	0.992	1.000	1.000	1.000
200	0.943	1.000	1.000	1.000	1.000	1.000	0.995	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000
400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: Numbers in the table denote the sizes of the $BCIPS^*$ *test*. Data are generated by: $y_{it} = \mu_{i,1}(1-\phi_i L)t + (1-\phi_i L)(\mu_i + \alpha_{i,1}\sin(2\pi kt/T) + \alpha_{i,2}\cos(2\pi kt/T)) + \phi_i y_{it-1} + \gamma_i f_t + \varepsilon_{it}$, for *i=1,…,N and t=1,…,T,* where y_{i0}, $f_t \sim$ i.i.d.$N(0.1)$; μ_i, $\mu_{i,1} \sim$ i.i.d.$U(0, 0.02)$ and $\gamma_i \sim$ i.i.d.$U[-1,3]$ (high cross-sectional dependence); $\alpha_{i,1}, \alpha_{i,2} \sim$ i.i.d.$U[1,2]$ (high amplitude); $\varepsilon_{it} \sim$ i.i.d.$N(0,\sigma_i^2)$ with $\sigma_i^2 \sim$ i.i.d.$U[0.5,1.5]$. Size (under the null $\phi_i = 1$) and power (under the alternative $\phi_i \sim$ i.i.d.$U[0.85, 0.95]$) are computed at the 5% nominal level based on the *BCADF(0)* regression : $\Delta y_{it} = c_{i,0} + \tau \cdot t + c_{i,1}\Delta\sin(2k\pi t/T) + c_{i,2}\Delta\cos(2k\pi t/T) + c_{i,3}\bar{y}_{t-1} + c_{i,4}\Delta\bar{y}_t + b_i y_{it-1} + e_{it}$, *i=1,…,N, t=1,…,T, k*=1,…,3, where $\bar{y}_t = N^{-1}\sum_{j=1}^{N} y_{jt}$, $\Delta\bar{y}_t = \bar{y}_t - \bar{y}_{t-1}$. $BCIPS^*(N,T) = N^{-1}\sum_{i=1}^{N} t_i^*(N,T)$, where $t_i^*(N,T)$ is the truncated t-statistic described in (23).

Table 6. Size and power of the panel unit-root test: high cross-sectional dependence, breaks with high amplitude, serially correlated errors (Trend case)

	k =1						k =2						k =3					
T/N	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200
							Size: Positive residual serial correlations											
30	0.063	0.062	0.066	0.053	0.053	0.058	0.048	0.031	0.033	0.030	0.027	0.026	0.020	0.011	0.009	0.006	0.002	0.002
50	0.057	0.061	0.060	0.061	0.073	0.069	0.050	0.043	0.039	0.039	0.032	0.033	0.025	0.020	0.014	0.012	0.007	0.004
100	0.056	0.069	0.059	0.057	0.057	0.063	0.059	0.049	0.044	0.046	0.045	0.043	0.043	0.039	0.024	0.022	0.021	0.018
200	0.057	0.048	0.051	0.051	0.057	0.055	0.052	0.051	0.047	0.044	0.044	0.041	0.047	0.038	0.040	0.034	0.032	0.029
400	0.049	0.053	0.056	0.052	0.052	0.051	0.057	0.050	0.052	0.049	0.053	0.045	0.044	0.049	0.040	0.048	0.040	0.045
							Size: Negative residual serial correlations											
30	0.053	0.078	0.062	0.066	0.066	0.075	0.064	0.079	0.065	0.069	0.063	0.073	0.047	0.049	0.042	0.033	0.037	0.036
50	0.054	0.049	0.057	0.055	0.059	0.064	0.053	0.055	0.064	0.062	0.058	0.067	0.044	0.045	0.041	0.036	0.035	0.037
100	0.067	0.084	0.053	0.053	0.062	0.059	0.063	0.080	0.054	0.051	0.055	0.053	0.062	0.069	0.048	0.044	0.040	0.042
200	0.067	0.046	0.051	0.053	0.054	0.051	0.065	0.053	0.054	0.049	0.049	0.052	0.056	0.052	0.056	0.050	0.047	0.042
400	0.048	0.044	0.052	0.046	0.053	0.052	0.060	0.050	0.050	0.047	0.053	0.053	0.056	0.051	0.052	0.050	0.052	0.045
							Power: Positive residual serial correlations											
30	0.069	0.069	0.073	0.063	0.066	0.072	0.069	0.053	0.047	0.047	0.040	0.035	0.032	0.022	0.017	0.011	0.005	0.005
50	0.073	0.095	0.096	0.112	0.130	0.140	0.085	0.114	0.102	0.130	0.119	0.137	0.057	0.089	0.067	0.096	0.069	0.088
100	0.236	0.408	0.491	0.555	0.754	0.904	0.419	0.671	0.749	0.803	0.911	0.973	0.469	0.761	0.861	0.920	0.983	0.999
200	0.876	0.996	1.000	1.000	1.000	1.000	0.985	1.000	1.000	1.000	1.000	1.000	0.996	1.000	1.000	1.000	1.000	1.000
400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
							Power: Negative residual serial correlations											
30	0.061	0.087	0.070	0.077	0.071	0.087	0.069	0.102	0.090	0.087	0.073	0.097	0.055	0.088	0.073	0.064	0.062	0.068
50	0.070	0.081	0.097	0.106	0.124	0.124	0.095	0.124	0.152	0.172	0.176	0.182	0.112	0.150	0.187	0.224	0.270	0.310
100	0.285	0.338	0.498	0.681	0.905	0.982	0.521	0.540	0.766	0.920	0.993	1.000	0.639	0.664	0.903	0.990	1.000	1.000
200	0.984	1.000	1.000	1.000	1.000	1.000	0.999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
400	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Notes: Numbers in the table denote the sizes or powers of the $BCIPS^*$ *test*. Data are generated by: $y_{it} = \mu_{i,1}(1-\phi_i L)t + (1-\phi_i L)(\mu_i + \alpha_{i,1}\sin(2\pi kt/T) + \alpha_{i,2}\cos(2\pi kt/T)) + \phi_i y_{it-1} + \gamma_i f_t + \varepsilon_{it}$ and $\varepsilon_{it} = \rho_i \varepsilon_{it-1} + e_{it}$, for *i=1,…,N*, *t=1,…,T and k=1,…,3*. See Notes on Table 5 for parameter and initial values setting.

Table 7. Size of the panel unit-root test allowing for smoothing breaks when k is unknown

	k =1						k =2						k =3					
T/N	10	20	30	50	100	200	10	20	30	50	100	200	10	20	30	50	100	200
							The intercept case											
30	0.072	0.059	0.060	0.050	0.047	0.053	0.080	0.073	0.066	0.064	0.062	0.059	0.059	0.047	0.059	0.051	0.049	0.049
50	0.071	0.064	0.049	0.049	0.045	0.051	0.074	0.075	0.057	0.060	0.052	0.050	0.062	0.057	0.049	0.050	0.046	0.057
100	0.074	0.070	0.056	0.047	0.052	0.046	0.073	0.070	0.051	0.052	0.060	0.048	0.066	0.057	0.053	0.049	0.054	0.053
200	0.076	0.064	0.055	0.055	0.047	0.052	0.065	0.053	0.036	0.049	0.052	0.051	0.059	0.049	0.045	0.050	0.049	0.052
400	0.080	0.062	0.063	0.058	0.051	0.052	0.075	0.050	0.042	0.046	0.041	0.042	0.061	0.050	0.038	0.046	0.046	0.044
							The linear trend case											
30	0.102	0.067	0.066	0.062	0.052	0.054	0.116	0.093	0.088	0.099	0.090	0.103	0.086	0.065	0.061	0.059	0.049	0.055
50	0.102	0.084	0.063	0.062	0.066	0.049	0.107	0.102	0.070	0.078	0.079	0.072	0.079	0.062	0.056	0.057	0.055	0.047
100	0.100	0.081	0.074	0.061	0.054	0.055	0.089	0.077	0.068	0.060	0.071	0.067	0.072	0.067	0.054	0.053	0.054	0.059
200	0.086	0.072	0.066	0.059	0.060	0.054	0.076	0.061	0.056	0.057	0.061	0.056	0.065	0.051	0.052	0.051	0.053	0.054
400	0.098	0.086	0.080	0.062	0.063	0.051	0.085	0.066	0.058	0.059	0.065	0.055	0.066	0.055	0.035	0.053	0.049	0.053

Notes: the number in the table is the size of the $BCIPS^*$ statistic in which the frequency component (k) in the Fourier function is selected based on the minimum of the sum of SSR rule described in the paper. The $BCIPS^*$ statistic is defined as $BCIPS^*(N,T) = N^{-1}\sum_{i=1}^{N} t_i^*(N,T)$, where $t_i^*(N,T)$ is the truncated t-statistic described in (23).

Table 8. *BCIPS** and *CIPS** tests of real exchange rates

	p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=8	p=9	p=10	p=11	p=12
					Monthly data, *N*=60, *T*=448, $\hat{k}$=1							
Ave(ρ)	0.168, #	0.168, #	0.167, #	0.168, #	0.168, #	0.168, #	0.169, #	0.169, #	0.170, #	0.170, #	0.171, #	0.172, #
Ave(c1)	-0.014	-0.014	-0.007	-0.008	-0.002	-0.028	-0.030	-0.029	-0.027	-0.027	-0.019	-0.021
Ave(c2)	-0.013	-0.004	-0.012	0.013	-0.007	0.048	0.046	0.021	0.007	-0.009	-0.007	-0.007
*BCIPS**	-3.749*	-3.535*	-3.608*	-3.493*	-3.693*	-3.502*	-3.513*	-3.582*	-3.661*	-3.789*	-3.663*	-3.679*
*CIPS**	-2.808*	-2.632*	-2.678*	-2.578	-2.698*	-2.529	-2.515	-2.549	-2.594	-2.673*	-2.565	-2.544
*A*F	3.796*	3.554*	3.686*	3.613*	4.072*	3.785*	3.804*	3.983*	4.175*	4.411*	4.293*	4.404*
					Monthly data, *N*=39, *T*=448, $\hat{k}$=1							
Ave(ρ)	0.280, #	0.278, #	0.278, #	0.279, #	0.278, #	0.278, #	0.278, #	0.279, #	0.280, #	0.281, #	0.282, #	0.281, #
Ave(c1)	-0.050	-0.050	-0.033	-0.039	-0.040	-0.042	-0.035	-0.028	-0.031	-0.032	-0.028	-0.028
Ave(c2)	0.000	-0.003	-0.002	0.011	0.011	0.014	-0.003	-0.013	-0.022	-0.025	-0.026	-0.041
*BCIPS**	-3.669*	-3.470*	-3.546*	-3.401	-3.417	-3.525*	-3.606*	-3.754*	-3.790*	-3.831*	-3.735*	-3.783*
*CIPS**	-2.525	-2.361	-2.415	-2.274	-2.282	-2.328	-2.354	-2.447	-2.451	-2.466	-2.388	-2.372
*A*F	4.167*	3.916*	--	--	3.915*	4.205*	4.440*	4.772*	4.887*	5.048*	4.861*	5.032*
	Quarterly data, N=60, T=149, $\hat{k}$=1							Quarterly data, N=39, T=149, $\hat{k}$=1				
	p=1	p=2	p=3	p=4				p=1	p=2	p=3	p=4	
Ave(ρ)	0.190, #	0.191, #	0.196, #	0.199, #				0.302, #	0.302, #	0.307, #	0.306, #	
Ave(c1)	-0.020	-0.023	-0.019	-0.021				-0.028	-0.024	-0.017	-0.021	
Ave(c2)	0.021	0.008	-0.036	-0.007				0.007	-0.013	-0.041	-0.043	
*BCIPS**	-3.781*	-3.458*	-3.808*	-3.588*				-3.607*	-3.567*	-3.819*	-3.626*	
*CIPS**	-2.735*	-2.421	-2.627*	-2.414				-2.423	-2.308	-2.435	-2.228	
*A*F	4.171*	3.878*	4.613*	4.440*				4.226*	4.401*	5.056*	4.819*	

Notes: k is the single-frequency component in the Fourier function and $\hat{k}$ is the estimated k. '#' indicates that the cross-sectional dependence test of Pesaran (2004) rejects the hypothesis of no cross-sectional dependence of residuals obtained from the ADF regression equation augmented with a Fourier function. Ave(ρ) is the mean of the pair-wise, cross-sectional correlation coefficients of the residuals from the above regression. Ave(c1) and Ave(c2) are the cross-sectional average of the estimated coefficients on the sine and cosine terms in the Fourier function. CIPS* is the panel unit-root test of Pesaran (2007) and BCIPS* is the panel unit-root test proposed by the paper. '*'' denotes significance at the 5% level. The AF statistic is the average of the F statistics across individuals in the panel. '--" indicates that a statistic is not constructed.